Symmetric bi derivations in GK algebra

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Abstract: In this paper, we introduce the concept of symmetric bi derivation of GK algebra and also obtain some properties about this concept.

I.INTRODUCTION

Gy.Maska[2] was introduced the concept of symmetric biderivation,and later J.Vukman[4] was proved few results of symmetric bi-derivation on prime and semi prime rings. In 2011[3], sabahatt in Ilbira.Alev Firat and young Bae Jun was introduced the notion of symmetric bi derivation of BCI algebra. The authors[1] T.Ganeshkumar and M.chandramouleeswaran have introduced the concept of symmetric bi derivation of TM algebra. Afterwards few authors have applied the concept of symmetric bi derivation in their papers.Induced by these works, in our paper, we introduce the concept of symmetric bi derivation on GK algebra and also discuss about some interesting properties.

II.SYMMETRIC BI DERIVATION OF GK ALGEBRA

Definition:2.1 Let $(X,*,1)$ be a GK algebra. A mapping $\mathbb{N}$:XxX→X is said to be a left right symmetric bi derivation(simply LR symmetric bi derivation) of X, if it is satisfying the following identity

$\mathbb{N}(x*y,z) = (\mathbb{N}(x,z)*y) \wedge (x*\mathbb{N}(y,z))$ for x,y,z$\in X$.

Definition:2.2 Let $(X,*,1)$ be a GK algebra. A mapping $\mathbb{N}$:XxX→X is said to be a right left symmetric bi derivation(simply RL symmetric bi derivation) of X, if it is satisfying the following identity

$\mathbb{N}(x,y*z) = (\mathbb{N}(x,y)*z) \wedge (y*\mathbb{N}(x,z))$ for x,y,z$\in X$.

In general , if $\mathbb{N}$ is both LR and RL symmetric bi derivation then it is called as $\mathbb{N}$ is symmetric bi derivation.

Definition:2.3 Let X be a GK algebra. A map $\mathbb{N}$:XxX→X is said to be symmetric if $\mathbb{N}(x,y) = \mathbb{N}(y,x)$ ∀ pairs of x,y$\in X$.

Definition:2.4 Let X be a GK algebra and the mapping $\mathbb{N}$:XxX→X be a symmetric mapping. A map $\eta: X \to X$ be defined as $\eta(x) = \mathbb{N}(x,x)$ is called trace of $\mathbb{N}$.

Example:2.5 Consider the following cayley's table for GK algebra

$*$	1	2	3	4
1	1	2	3	4
2	2	1	4	3
3	3	4	1	2
4	4	3	2	1

Define a mapping $\mathbb{N}: X \times X \to X$ by

$$\mathbb{N}(x,y) = \begin{cases} 1, & (x,y) = (1,1),(2,2),(3,3),(4,4) \\ 2, & (x,y) = (1,2),(2,1),(3,4),(4,3) \\ 3, & (x,y) = (1,3),(2,4),(3,1),(4,2) \\ 4, & (x,y) = (1,4),(2,3),(3,2),(4,1) \end{cases}$$

From this $\mathbb{N}$ is symmetric bi derivation of X.

Remark:2.6 In above example, $\mathbb{N}(x,x) = \{1 \ \ when \ \ x = 1,2,3,4$ is called trace of $\mathbb{N}$.

Definition:2.7 Let X be a GK algebra. The map $\mathbb{N}: X \times X \to X$ be a symmetric mapping. $\mathbb{N}$ is called component wise regular if $\mathbb{N}(x,1) = \mathbb{N}(1,x) = 1 \ for\ some\ x \in X$. In specific if $\mathbb{N}(1,1) = \eta(1) = 1$ then $\mathbb{N}$ *is called* $\eta - regular$.

Proposition:2.8

Let $(X,*,1)$ be a GK algebra. Let $\mathbb{N}$ be an LR symmetric bi derivation on X. Then the following holds

- (i) $\mathbb{N}(x,y) = \mathbb{N}(x,y) \wedge \left(x * \mathbb{N}(1,y)\right) \ for\ all\ \ x,y \in X.$
- (ii) $\mathbb{N}(1,x) = \eta(x) * x$ where η is the trace of $\mathbb{N}$.
- (iii) $\mathbb{N}(1,y) = \mathbb{N}(x,y) * x \ \ \forall \ x,y \in X.$
- (iv) $\mathbb{N}(y,1) = \mathbb{N}(y,1) \wedge y \ \forall\ y\ in\ X\ if\ \ \mathbb{N}\ \ is\ \eta - regular\ .$
- (v) $\mathbb{N}(y,1) = 1 \ \ \forall\ y\ in\ X\ if\ \ \mathbb{N}\ \ is\ component\ wise\ regular$

Proof:

(i) Let us consider x,y in X
By the definition of LR bi symmetric bi derivation,
We have,
$\mathbb{N}(\boldsymbol{x},\boldsymbol{y}) = \mathbb{N}(x * 1, y)$
$=(\mathbb{N}(x,y) * 1) \wedge (x * \mathbb{N}(1,y))$

By axiom (ii) of GK algebra

$= (\mathbb{N}(x, y)) \wedge (x * \mathbb{N}(1, y))$

(ii) Let x, y in X

Now,

$\mathbb{N}(\mathbf{1}, \boldsymbol{x}) = \mathbb{N}(x * x, x)$

$= (\mathbb{N}(x, x) * x) \wedge (x * \mathbb{N}(\mathrm{x}, x))$

$= (\eta(x) * x) \wedge \left(x * \eta(x)\right)$

$=\left(x * \eta(x)\right) * (\left(x * \eta(x)\right) * (\eta(x) * x))$

$= (\eta(x) * x)$

(iii) Let x, y in X

We have,

$\mathbb{N}(\mathbf{1}, \boldsymbol{y}) = \mathbb{N}(x * x, y)$

$= (\mathbb{N}(x, y) * x) \wedge (x * \mathbb{N}(\mathrm{x}, y))$

$= (x * \mathbb{N}(\mathrm{x}, y)) * ((x * \mathbb{N}(\mathrm{x}, y) * (\mathbb{N}(\mathrm{x}, y) * x))$

$= \mathbb{N}(\mathrm{x}, y) * x$

(iv) Let x, y in X

$\mathbb{N}(\boldsymbol{y}, \mathbf{1}) = \mathbb{N}(y * 1,1)$

$= (\mathbb{N}(y, 1) * 1) \wedge (y * \mathbb{N}(1,1))$

$= (\mathbb{N}(\mathrm{y}, 1)) \wedge (y * \eta(1))$

$= \mathbb{N}(\mathrm{y}, 1) \wedge (y * 1)$

$= \mathbb{N}(\mathrm{y}, 1) \wedge y$

(v) Let x, y in X

$\mathbb{N}(\boldsymbol{y}, \mathbf{1}) = \mathbb{N}(y * 1,1)$

$= (\mathbb{N}(y, 1) * 1) \wedge (y * \mathbb{N}(1,1))$

$= (\mathbb{N}(\mathrm{y}, 1)) \wedge (y * \eta(1))$

$= \mathbb{N}(\mathrm{y}, 1) \wedge (y * 1)$

$= \mathbb{N}(\mathrm{y}, 1) \wedge y$

$= 1 \wedge y$

$= 1$ since $x \wedge y = x$.

Proposition:2.9 Let $(X,*,1)$ be a GK algebra. Let $\mathbb{N}$ be an RL symmetric bi derivation on X. Then the following holds

(i) $\mathbb{N}(x, y) = \mathbb{N}(x, y) \wedge \left(x * \mathbb{N}(1, y)\right)$ *for all* $x, y \in X$.

(ii) $\mathbb{N}(1, x) = \eta(x) * x$ where η is the trace of $\mathbb{N}$.

(iii) $\mathbb{N}(1, y) = \mathbb{N}(x, y) * x \ \forall \ x, y \in X$.

(iv) $\mathbb{N}(y, 1) = \mathbb{N}(y, 1) \wedge y \, \forall \, y \; in \; X \; if \; \mathbb{N} \; is \; \eta - regular$.

(v) $\mathbb{N}(y, 1) = 1 \ \forall \, y \; in \; X \; if \; \mathbb{N} \; is \; component \; wise \; regular$

Proof:

(i) Let us consider x,y in X

By the definition of RL bi symmetric bi derivation,

We have,

$\mathbb{N}(\boldsymbol{x}, \boldsymbol{y}) = \mathbb{N}(x, y * 1)$

$=(\mathbb{N}(x,y)*1)\wedge(y*\mathbb{N}(x,1))$

By axiom (ii) of GK algebra

$=(\mathbb{N}(x,y))\wedge(y*\mathbb{N}(x,1))$

$=\mathbb{N}(x,y)\wedge(y*1)$

$=\mathbb{N}(x,y)\wedge y$

(ii) Let x, y in X

Now,

$\mathbb{N}(\mathbf{1},\boldsymbol{x})=\mathbb{N}(x,x*x)$

$=(\mathbb{N}(x,x)*x)\wedge(x*\mathbb{N}(x,x))$

$=(\eta(x)*x)\wedge\left(x*\eta(x)\right)$

$=\left(x*\eta(x)\right)*(\left(x*\eta(x)\right)*(\eta(x)*x))$

$=(\eta(x)*x)$

(iii) Let x, y in X

We have,

$\mathbb{N}(\boldsymbol{y},\mathbf{1})=\mathbb{N}(y,x*x)$

$=(\mathbb{N}(y,x)*x)\wedge(x*\mathbb{N}(y,x))$

$=(x*\mathbb{N}(y,x))*((x*\mathbb{N}(y,x)*(\mathbb{N}(y,x)*x))$

$=\mathbb{N}(y,x)*x$

(iv) Let x, y in X

$\mathbb{N}(\mathbf{1},\boldsymbol{y})=\mathbb{N}(1,y*1)$

$=(\mathbb{N}(1,y)*1)\wedge(y*\mathbb{N}(1,1))$

$=(\mathbb{N}(1,y))\wedge(y*\eta(1))$

$=\mathbb{N}(1,y)\wedge(y*1)$

$=\mathbb{N}(1,y)\wedge y$

(v) Let x, y in X

$\mathbb{N}(\boldsymbol{y},\mathbf{1})=\mathbb{N}(y*1,1)$

$=(\mathbb{N}(y,1)*1)\wedge(y*\mathbb{N}(1,1))$

$=(\mathbb{N}(y,1))\wedge(y*\eta(1))$

$=\mathbb{N}(y,1)\wedge(y*1)$

$=\mathbb{N}(y,1)\wedge y$

$=1\wedge y$

$=y*(y*1)$

$=1.$

Proposition:2.10 Let X be the GK algebra and η be the trace of the LR symmetric bi derivation on X. Then

(i) $\eta(1)=\mathbb{N}(x,1)*x.$

(ii) $If\ \mathbb{N}(x,1)=\mathbb{N}(y,1)\ \ \forall\ x,y\ \in X\ then\ \eta\ is\ 1-1.$

(iii) η is regular iff $\mathbb{N}(x,1)=x.$

Proof:

(i) Let $x\in X$. We know that $x*x=1$

We have,

$\boldsymbol{\eta}(\mathbf{1}) = \mathbb{N}(1,1)$
$= \mathbb{N}(\boldsymbol{x} * \boldsymbol{x}, \mathbf{1})$
$= (\mathbb{N}(\boldsymbol{x}, \mathbf{1}) * \boldsymbol{x}) \wedge (\boldsymbol{x} * \mathbb{N}(\boldsymbol{x}, \mathbf{1}))$
$= (\mathbb{N}(\boldsymbol{x}, \mathbf{1}) * \boldsymbol{x})$

(ii) **Let** x, y ∈ X such that $\eta(x) = \eta(y)$.
We have,
$\eta(1) = \mathbb{N}(x, 1) * x$
and
$\eta(1) = \mathbb{N}(y, 1) * y$.
This implies that
$\mathbb{N}(x, 1) * x = \mathbb{N}(y, 1) * y$.
Since $\mathbb{N}(x, 1) = \mathbb{N}(y, 1)$ and by using cancellation law, we get
x=y.
Hence we get η is 1-1.

(iii) Let η be regular.
We have
$\eta(1) = \mathbb{N}(x, 1) * x$
Since η is regular
$\eta(1) = 1$ *implies* $\mathbb{N}(x, 1) * x = 1$.
By axiom (iii) of GK algebra we have $\mathbb{N}(x, 1) = x$
Conversely ,

Let $\mathbb{N}(x, 1) = x$ for some x in X.
$\Rightarrow \mathbb{N}(x, 1) * x = x * x$
$\Rightarrow \mathbb{N}(x, 1) * x = 1$
$\Rightarrow \eta(1) = 1$
Hence η is regular.

Proposition:2.11 Let X be the GK algebra and η be the trace of the RL symmetric bi derivation on X. Then

(i) $\eta(1) = \mathbb{N}(1, x) * x$.
(ii) $\eta(x) = \eta(x) \wedge (x * \mathbb{N}(x, 1))$
(iii) *If* $\mathbb{N}(1, x) = \mathbb{N}(1, y)\ \ \forall\ x, y \in X$ *then* η *is* $1 - 1$.
(iv) η is regular iff $\mathbb{N}(1, x) = x$.

Proof:

(i) Let $x \in X$. We know that $x * x = 1$
We have,
$\boldsymbol{\eta}(\mathbf{1}) = \mathbb{N}(1,1)$
$= \mathbb{N}(\mathbf{1}, \boldsymbol{x} * \boldsymbol{x})$
$= (\mathbb{N}(\mathbf{1}, \boldsymbol{x}) * \boldsymbol{x}) \wedge (\boldsymbol{x} * \mathbb{N}(\mathbf{1}, \boldsymbol{x}))$
$= (\mathbb{N}(\mathbf{1}, \boldsymbol{x}) * \boldsymbol{x})$

(ii) Let x in X

$\boldsymbol{\eta}(\boldsymbol{x}) = \mathbb{N}(x, x)$

$= \mathbb{N}(\boldsymbol{x}, \boldsymbol{x} * \mathbf{1})$

$= (\mathbb{N}(\boldsymbol{x}, \boldsymbol{x}) * \mathbf{1}) \wedge (\boldsymbol{x} * \mathbb{N}(\boldsymbol{x}, \mathbf{1}))$

$= (\eta(x) * \mathbf{1}) \wedge (\boldsymbol{x} * \mathbb{N}(\boldsymbol{x}, \mathbf{1}))$

$= \eta(x) \wedge (\boldsymbol{x} * \mathbb{N}(\boldsymbol{x}, \mathbf{1}))$

If it is component wise regular , we get $\eta(x) \wedge x$.

(iii) **Let** x, y ∈ X such that $\eta(x) = \eta(y)$.
We have,
$\eta(1) = \mathbb{N}(1, x) * x$
and
$\eta(1) = \mathbb{N}(1, y) * y$.
This implies that
$\mathbb{N}(1, x) * x = \mathbb{N}(1, y) * y$.
Since $\mathbb{N}(1, x) = \mathbb{N}(1, y)$ and by using cancellation law, we get
x=y.
Hence we get η is 1-1.

(iv) Let η be regular.
We have
$\eta(1) = \mathbb{N}(1, x) * x$
Since η is regular
$\eta(1) = 1$ *implies* $\mathbb{N}(1, x) * x = 1$.
By axiom (iii) of GK algebra we have $\mathbb{N}(1, x) = x$

Conversely ,
Let $\mathbb{N}(1, x) = x$ for some x in X.
$\Rightarrow \mathbb{N}(1, x) * x = x * x$
$\Rightarrow \mathbb{N}(1, x) * x = 1$
$\Rightarrow \eta(1) = 1$
Hence η *is regular.*

References

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