Eccentricity indices of polyhex nanotubes $TUAC_6(p, q)$ and $TUZC_6(p, q)$

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Abstract: In this paper, we determine Harmonic eccentricity index, Eccentricity connectivity index and some Zagreb eccentric indices of armchair polyhex nanotubes $TUAC_6$ and zigzag polyhex nanotubes $TUZC_6$.

Keyword: Eccentricity Index, molecular graph, armchair polyhex nanotube $TUAC_6$ and zigzag polyhex nanotube $TUZC_6$.

1.Introduction

Let G be a graph with n vertices and m edges with vertex set V(G) and edge set E(G). The molecular graphs considered in this paper are finite, connected, loopless, and without multiple edges. The degree of a vertex $u \in V(G)$ is denoted by $d_u(G)$ and is the number of vertices that are adjacent to u. The edge connecting the vertices u and v is denoted by uv. The airmchair polyhex nanotubes G = $TUAC_6$(p, q) is a graph where p and q denote the number of hexagons in the each row and number of rows in the molecular graph of the 2D-lattices of $TUAC_6(p, q)$, respectively. Thus the number of vertices/atoms in this nanotube is equal to $|V(TUAC_6[p, q])| = 4pq$, the number of edges/ bonds is $|E(TUAC_6[p, q])| = 6pq – 2p$.

The zigzag polyhex nanotube G = $TUZC_6(p, q)$ is a graph where p and q denote the number of hexagons in the each row and q is the number of zigzag lines in the molecular graph of the 2D-lattices of $TUZC_6[p, q]$, respectively. Thus the number of vertices/atoms in this nanotube is equal to $|V(TUZC_6[p, q])| = 2pq$, the number of edges/ bonds is $|E(TUZC_6[p, q])| = 3pq – p$.

Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, $e(v) = \max\{d(u, v); u \in V\}$. The radius r(G) is the minimum eccentricity of the vertices, whereas the diameter diam(G) is the maximum eccentricity.

In[2], Bhanumathi and Easu Julia Rani., introduced the Harmonic eccentric index and it is defined as $HEI(G) = \sum_{uv \in E(G)} \frac{2}{e_u + e_v}$,where e_u denote the eccentricity of the vertex u.

Ghorbani et al.[4] and Vukicevic et al. [11], introduced first and second Zagreb eccentric indices, which are defined as

$$E_1(G) = \sum_{u \in V(G)} e_u^2$$

$$E_2(G) = \sum_{uv \in E(G)} e_u e_v$$

Sharma et al. introduced the eccentricity connectivity index of a graph (Sharma, Goswami, & Madan, 1997), where they defined it for a graph with n vertices and m edges, as

$$\varepsilon^{c}(G) = \sum_{u \in V(G)} d_u e_u$$

The first and second multiplicative Zagreb eccentric indices of the connected graph G were introduced by De[3] and are defined as

$$\Pi E_1(G) = \prod_{u \in V(G)} e_u^2$$

$$\Pi E_2(G) = \prod_{uv \in E(G)} e_u e_v$$

2. Eccentricity indices of Armchair polyhex nanotube $TUAC_6(p, q)$

In this section, we evaluate Zagreb eccentric indices, Eccentricity connectivity index, Multiplicative Zagreb eccentric indices, Harmonic eccentricity index of Armchair polyhex nanotube.

Let G = $TUAC_6(p, q)$ be an armchair polyhex nanotube, where p is the number of hexagons in each row and q is the number of rows in the molecular graph G. The graph $TUAC_6(p, q)$ has 4pq vertices and 6pq – 2p edges. Number of vertices and eccentricity of vertices in G are given in Table 2.1. Number of edges and eccentricity of end vertices in G are given in Table 2.2.

Sl.No	Number of vertices	Eccentricity of vertices e_u	Degree of vertices d_u
1	2p	p + 2q – 1	2
2	2p	p + 2q – 2	3
3	2p	p + 2q – 3	3
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q – 1	2p	p + q + 1	3
q	2p	p + q	3
q + 1	2p	p + q	3
q + 2	2p	p + q +1	3
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q + 1 + q – 2	2p	p + 2q – 2	3
q + 1 + q – 1 = 2q	2p	p + 2q – 1	2

Table 2.1

Sl.No	Number of edges uv	Eccentricity of end vertices (e_u, e_v)
1	p 2p	(p + 2q – 1, p + 2q – 1) (p + 2q – 1, p + 2q – 2)
2	p 2p	(p + 2q – 2, p + 2q – 2) (p + 2q – 2, p + 2q – 3)
3	p 2p	(p + 2q – 3, p + 2q – 3) (p + 2q – 3, p + 2q – 4)
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q	p 2p	(p + q , p + q) (p + q, p + q)
q + 1	p 2p	(p + q , p + q) (p + q , p + q + 1)
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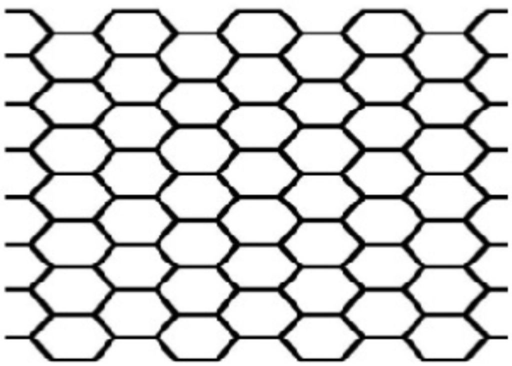
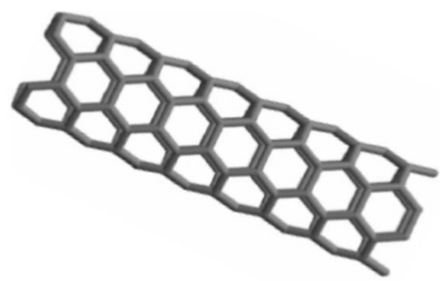
2q – 1	p 2p	(p + 2q – 2, p + 2q – 2) (p + 2q – 2, p + 2q – 1)
2q	p	(p + 2q – 1, p + 2q – 1)

Table 2.2

Example: Let G = $TUAC_6(4, 8)$. We determine the Harmonic eccentricity index, first and second Zagreb indices, eccentricity connectivity index and first and second multiplicative Zagreb indices for this nanotubes.





(a) **(b)**

Fig (a) The 2-dimensional lattice (b) $TUAC_6$ nanotube of $TUAC_6(4, 8)$ nanotube

Sl.No	Number of vertices	Eccentricity of vertices e_u	degree of vertices d_u	Sl.No	Number of vertices	Eccentricity of vertices e_u	degree of vertices d_u
1	8	19	2	9	8	12	3
2	8	18	3	10	8	13	3
3	8	17	3	11	8	14	3
4	8	16	3	12	8	15	3
5	8	15	3	13	8	16	3
6	8	14	3	14	8	17	3
7	8	13	3	15	8	18	3
8	8	12	3	16	8	19	2

Table 2.3

Sl.No	Number of edges uv	Eccentricity of end vertices (e_u, e_v)
1	4 8	(19, 19) (19, 18)
2	4 8	(18, 18) (18, 17)
3	4 8	(17,17) (17,16)

4	4 8	(16, 16) (16, 15)
5	4 8	(15, 15) (15,14)
6	4 8	(14, 14) (14, 13)
7	4 8	(13, 13) (13,12)
8	4 8	(12,12) (12, 12)
9	4 8	(12, 12) (12, 13)
10	4 8	(13, 13) (13, 14)
11	4 8	(14, 14) (14, 15)
12	4 8	(15, 15) (15, 16)
13	4 8	(16, 16) (16, 17)
14	4 8	(17, 17) (17, 18)
15	4 8	(18, 18) (18, 19)
16	4	(19, 19)

Table 2.4

(i) $HE_1(G)$ =

$$2\left\{\begin{array}{l}4\left(\frac{2}{19+19}\right)+8\left(\frac{2}{19+18}\right)+4\left(\frac{2}{18+18}\right)+8\left(\frac{2}{18+17}\right)+4\left(\frac{2}{17+17}\right)+8\left(\frac{2}{17+16}\right)+4\left(\frac{2}{16+16}\right)+8\left(\frac{2}{16+15}\right)\\+4\left(\frac{2}{15+15}\right)+8\left(\frac{2}{15+14}\right)+4\left(\frac{2}{14+14}\right)+8\left(\frac{2}{14+13}\right)+4\left(\frac{2}{13+13}\right)+8\left(\frac{2}{13+12}\right)+4\left(\frac{2}{12+12}\right)\end{array}\right\}$$

$$+8\left(\frac{2}{12+12}\right)$$

= 2{ 0.2105 + 0.4324 + 0.2222 + 0.4571 +0.2353 + 0.4849 + 0.25 + 0.5161 + 0.2667 + 0.5517 + 0.2857 +0.5926 + 0.3077 + 0.64 + 0.3333} + 0.6667 = 2(57862) + 0.6667 = 12.2391.

(ii) $E_1(G)$ = 31424 (iii) $E_2(G)$ = 44192 (iii) ε^c (G) = 5648 (iv) $\Pi E_1(G) = (1.5581085210 \times 10^{26})^2$

(vi) $\Pi E_2(G) = (5.19912914 \times 10^{46})^2(1152)$

Theorem 2.1: Harmonic eccentricity index of G = $TUAC_6$(p, q) is $HE_1(G)$ = $2\sum_{k=0}^{q-1} p\frac{1}{p+q+k}$ + $2\sum_{k=1,3,.5,..}^{2q-3} 2p\frac{2}{2p+2q+k}$ $+ 2p\left(\frac{1}{p+q}\right)$

Proof: $HE_1(G) = \sum_{uv \in E(G)} \frac{2}{e_u + e_v}$

$$= p\frac{2}{p+2q-1+p+2q-1} + 2p\frac{2}{p+2q-1+p+2q-2} + p\frac{2}{p+2q-2+p+2q-2} + \ldots +$$

$$p\frac{2}{p+q+p+q} + 2p\frac{2}{p+q+p+q} + p\frac{2}{p+q+p+q} + \ldots + p\frac{2}{p+2q-2+p+2q-2} +$$

$$2p\frac{2}{p+2q-2+p+2q-1} + p\frac{2}{p+2q-1+p+2q-1}$$

$$= p\frac{2}{2p+4q-2} + 2p\frac{2}{2p+4q-3} + p\frac{2}{2p+4q-4} + \ldots + p\frac{2}{2p+2q} + 2p\frac{2}{2p+2q} +$$

$$p\frac{2}{2p+2q} + \ldots + p\frac{2}{2p+4q-4} + 2p\frac{2}{2p+4q-3} + p\frac{2}{2p+4q-2}$$

$$= 2\sum_{k=0}^{q-1} p\frac{1}{p+q+k} + 2\sum_{k=1,3,5,..}^{2q-3} 2p\frac{2}{2p+2q+k} + 2p\left(\frac{1}{p+q}\right)$$

Theorem 2.2: Zagreb eccentric index of G = $TUAC_6(p, q)$ is $E_1(G) = 2\sum_{k=0}^{q-1} 2p(p+q+k)^2$

Proof: $E_1(G) = \sum_{u \in V(G)} e_u^2$

$$= 2p(p+2q-1)^2 + 2p(p+2q-2)^2 + 2p(p+2q-3)^2 + \ldots + 2p(p+q)^2 + 2p(p+q)^2 +$$

$$2p(p+q+1)^2 + \ldots + 2p(p+2q-2)^2 + 2p(p+2q-1)^2$$

$$= 2\sum_{k=0}^{q-1} 2p(p+q+k)^2$$

Theorem 2.3: Zagreb eccentric index of G = $TUAC_6(p, q)$ is $E_2(G) = 2\sum_{k=0}^{q-1} p(p+q+k)^2 + 2\sum_{k=1}^{q-1} 2p((p+q+k)(p+q+(k-1)) + 2p(p+q)^2$

Proof: $E_2(G) = \sum_{uv \in E(G)} e_u e_v$

$$= \{p((p+2q-1)(p+2q-1))\} + \{2p((p+2q-1)(p+2q-2))\} + \{p((p+2q-2)(p+2q-2))\} +$$

$$\{2p((p+2q-2)(p+2q-3))\} + \ldots + \{p((p+q)(p+q))\} + \{2p((p+q)(p+q))\} + \{p((p+q)(p+q))\} + \ldots + \{p((p+2q-2)(p+2q-2))\} + \{2p((p+2q-2)(p+2q-1)\} + \{p((p+2q-1)(p+2q-1))\}$$

= p(p + 2q – 1)^2 + 2p ((p + 2q – 1) (p + 2q – 2)) + p(p + 2q – 2)^2 + 2p((p + 2q – 2) (p + 2q – 3)) +…..+ p(p + q)^2 + 2p((p + q)(p + q)) + p(p + q)^2 +…..+ p(p + 2q – 2)^2 + 2p((p + 2q – 2) (p + 2q – 1) + p(p + 2q – 1)^2

$$= 2\sum_{k=0}^{q-1} p(p+q+k)^2 + 2\sum_{k=1}^{q-1} 2p((p+q+k)(p+q+(k-1)) + 2p(p+q)^2$$

Theorem 2.4: Eccentricity connectivity index of G = TUAC$_6$(p, q) is ε^c (G) = $2\sum_{k=0}^{q-2} 6p(p+q+k)+$ 8p(p + 2q – 1)

Proof: ε^c (G) = $\sum_{u\in V(G)} d_u e_u$

= 2p(2(p + 2q – 1)) +2p(3(p + 2q – 2)) +2p(3(p + 2q – 3)) +…..+ 2p(3(p + q)) +2p(3(p + q)) +2p((p + q + 1)) +2p(3(p + 2q – 2)) + 2p(2(p + 2q – 1))

= 4p(p + 2q – 1) +6p(p + 2q – 2) +6p(p + 2q – 3) +…..+ 6p(p + q) +6p(p + q) +6p(p + q + 1) +6p(p + 2q – 2) + 4p(p + 2q – 1)

$$= 2\sum_{k=0}^{q-2} 6p(p+q+k) + 8p(p + 2q - 1)$$

Theorem 2.5: First Multiplicative Zagreb eccentric index of G = TUAC$_6$(p, q) is ΠE$_1$(G) = $\prod_{k=0}^{q-1}\left(2p(p+q+k)^2\right)^2$

Proof: ΠE$_1$(G) = $\prod_{u\in V(G)} e_u^2$

= (2p(p + 2q – 1)^2)(2p(p + 2q – 2)^2) (2p(p + 2q – 3)^2)…..(2p(p + q)^2) (2p(p + q)^2)

(2p(p + q + 1)^2)…..(2p(p + 2q – 2)^2) (2p(p + 2q – 1)^2)

$$= \prod_{k=0}^{q-1}\left(2p(p+q+k)^2\right)^2$$

Theorem 2.6: Second Multiplicative Zagreb eccentric index of G = TUAC$_6$(p, q) is ΠE$_2$(G) = $\prod_{k=0}^{q-1}\left(p(p+q+k)^2\right)^2 \prod_{k=1}^{q-1}\left(2p((p+q+k)(p+q+(k-1)))\right)^2 .2p(p+q)^2$

Proof: ΠE$_2$(G) = $\prod_{uv\in E(G)} e_u e_v$

= (p(p + 2q – 1)(p + 2q – 1)) (2p(p + 2q – 1)(p + 2q – 2)) (p(p + 2q – 2)(p + 2q – 2)) (2p(p + 2q – 2)(p + 2q – 3))… (p(p + q)(p + q)) (2p(p + q)(p + q)) (p(p + q)(p + q))… (p(p + 2q – 2)(p + 2q – 2)) (p(p + 2q – 2)(p + 2q – 1)) (p(p + 2q – 1)(p + 2q – 1))

$= (p(p + 2q - 1)^2)\ (2p(p + 2q - 1)\ (p + 2q - 2))\ (p(p + 2q - 2)^2)\ (2p(p + 2q - 2)\ (p + 2q - 3)) \ldots (p(p + q)^2)$
$(2p(p + q)\ (p + q))\ (p(p + q)^2)) \ldots (p(p + 2q - 2)^2)\ (2p(p + 2q - 2)\ (p + 2q - 1))(p(p + 2q - 1)^2)$

$$= \prod_{k=0}^{q-1} \left(p(p+q+k)^2\right)^2 \prod_{k=1}^{q-1} \left(2p((p+q+k)(p+q+(k-1)))\right)^2 .2p(p+q)^2$$

3. Eccentricity indices of Zigzag polyhex nanotube $TUZC_6(p, q)$

In this section, we evaluate Zagreb eccentric indices, Eccentricity connectivity index, Multiplicative Zagreb eccentric indices, Harmonic eccentricity index of Zigzag polyhex nanotube.

Let $G = TUZC_6(p, q)$ be a zigzag polyhex nanotube, where p is the number of hexagons in each row and q is the number of zigzag lines in the column of molecular graph of G. The graph $TUZC_6(p, q)$ has 2pq vertices and 3pq – p edges. Number of vertices and eccentricity of vertices in G are given in Table 3.1. Number of edges and eccentricity of end vertices of G are given in Table 3.2.

Sl.No	Number of vertices	Eccentricity of vertices e_u	degree of vertices d_u
1	p	(p + q/2) + (q/2 – 1)	2
2	p	(p + q/2) + (q/2 – 1)	3
3	p	(p + q/2) + (q/2 – 2)	3
4	p	(p + q/2) + (q/2 – 2)	3
5	p	(p + q/2) + (q/2 – 3)	3
6	p	(p + q/2) + (q/2 – 3)	3
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q/2 – 1	p	p + q/2	3
q/2	p	p + q/2	3
q/2 + 1	p	p + q/2 + 1	3
q/2 + 2	p	p + q/2 + 1	3
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q – 3	p	(p + q/2) + (q/2 – 2)	3
q – 2	p	(p + q/2) + (q/2 – 2)	3
q – 1	p	(p + q/2) + (q/2 – 1)	3
q	p	(p + q/2) + (q/2 – 1)	2

Table 3.1

Sl.No	Number of edges uv	Eccentricity of end vertices (e_u, e_v)
1	2p p	(p + q/2 + (q/2 – 1), p + q/2 + (q/2 – 1)) (p + q/2 + (q/2 – 1), p + q/2 + (q/2 – 2))
2	2p p	(p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 3))
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q / 2	2p p	(p + q/2, p + q/2) (p + q/2, p + q/2)
q / 2 +1	2p p	(p + q/2, p + q/2) (p + q/2, p + q/2 + 1)

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q – 1	2p p	(p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 2)) (p + q/2 + (q/2 – 2), p + q/2 + (q/2 – 1))
q	2p	(p + q/2 + (q/2 – 1), p + q/2 + (q/2 – 1))

Table 3.2

Example: Let G = $TUZC_6(8, 8)$. We determine the Harmonic eccentricity index, first and second Zagreb indices, eccentricity connectivity index and first and second multiplicative Zagreb indices for this nanotubes.

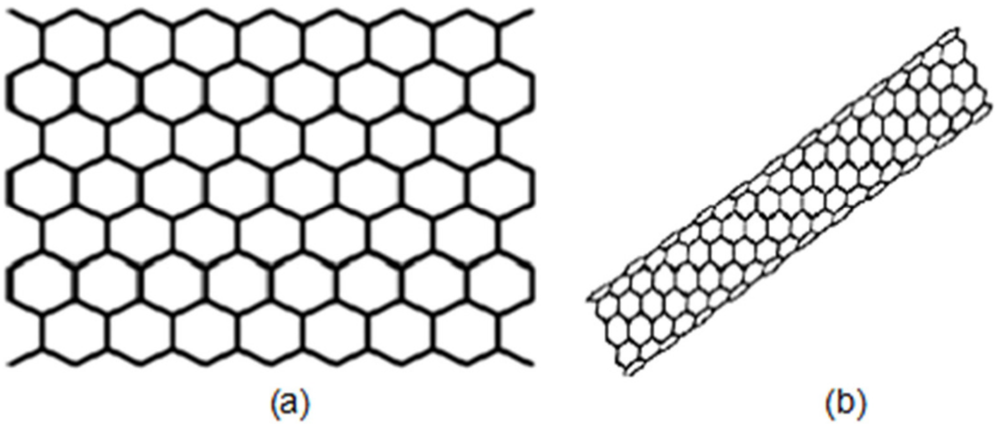

Fig (a) The 2-dimensional lattice (b) $TUZC_6$nanotube of $TUZC_6(8, 8)$ nanotube

Sl. No	Number of vertices	Eccentricity of vertices e_u	degree of vertices d_u	Sl. No	Number of vertices	Eccentricity of vertices e_u	degree of vertices d_u
1	8	15	2	9	8	12	3
2	8	15	3	10	8	12	3
3	8	14	3	11	8	13	3
4	8	14	3	12	8	13	3
5	8	13	3	13	8	14	3
6	8	13	3	14	8	14	3
7	8	12	3	15	8	15	3
8	8	12	3	16	8	15	2

Table 3.3

Sl.No	Number of edges uv	Eccentricity of end vertices (e_u, e_v)	Sl.No	Number of edges uv	Eccentricity of end vertices (e_u, e_v)
1	16 8	(15, 15) (15, 14)	5	16 8	(12, 12) (12, 13)
2	16 8	(14, 14) (14, 13)	6	16 8	(13, 13) (13, 14)
3	16 8	(13, 13) (13, 12)	7	16 8	(14, 14) (14, 15)
4	16 8	(12, 12) (12, 12)	8	16	(15, 15)

Table 3.4

(i) $HE_1(G)$ =

$$2\left\{16\left(\frac{2}{15+15}\right)+8\left(\frac{2}{15+14}\right)+16\left(\frac{2}{14+14}\right)+8\left(\frac{2}{14+13}\right)+16\left(\frac{2}{13+13}\right)+8\left(\frac{2}{13+12}\right)+16\left(\frac{2}{12+12}\right)\right\}$$

$+ 8\left(\frac{2}{12+12}\right)$ = 2{1.0667 + 0.5517 + 1.1429 + 0.5926 + 1.2308 +0.64 + 1.3333} + 0.6667 = 2(6.558) + 0.6667 = 13.116 + 0.6667 = 13.7827

(ii) $E_1(G) = 23488$ (iii) $E_2(G) = 33408$ (iv) $\varepsilon^c(G) = 4944$ (v) $\Pi E_1(G) = (1.932393056 \times 10^{25})^2$

(vi) $\Pi E_2(G) = (2.147103396 \times 10^{23})^2(1152)$.

Theorem 3.1: Harmonic eccentricity index of G = $TUZC_6(p, q)$ is $HE_1(G) = 2\sum_{k=0}^{q/2-1} 2p\frac{1}{p+q/2+k} + 2\sum_{k=1,3,5,..}^{q-3} p\frac{2}{2p+q+k} + p\left(\frac{1}{p+q/2}\right)$

Proof: $HE_1(G) = \sum_{uv\in E(G)} \frac{2}{e_u+e_v}$

$$= 2p\frac{2}{(p+q/2+(q/2-1)+p+q/2+(q/2-1))} + p\frac{2}{(p+q/2+(q/2-1)+p+q/2+(q/2-2))}$$

$$+ 2p\frac{2}{(p+q/2+(q/2-2)+p+q/2+(q/2-2))} +\ldots+ 2p\frac{2}{(p+q/2+p+q/2)} +$$

$$p\frac{2}{(p+q/2+p+q/2)} +\ldots+ p\frac{2}{(p+q/2+(q/2-2)+p+q/2+(q/2-1))} +$$

$$2p\frac{2}{(p+q/2+(q/2-1)+p+q/2+(q/2-1))}$$

$$= 2p\frac{2}{(2p+2q-2)} + p\frac{2}{(2p+2q-3)} + 2p\frac{2}{(2p+2q-4)} +\ldots+ 2p\frac{2}{(2p+q)} + p\frac{2}{(2p+q)} +\ldots+$$

$$2p\frac{2}{(2p+2q-4)} + p\frac{2}{(2p+2q-3)} + 2p\frac{2}{(2p+2q-2)}$$

$$= 2\sum_{k=0}^{q/2-1} 2p\frac{1}{p+q/2+k} + 2\sum_{k=1,3,5,..}^{q-3} p\frac{2}{2p+q+k} + p\left(\frac{1}{p+q/2}\right)$$

Theorem 3.2: Zagreb eccentric index of G = $TUZC_6(p, q)$ is $E_1(G) = 2\sum_{k=0}^{q/2-1} 2p(p+q/2+k)^2$

Proof: $E_1(G) = \sum_{u\in V(G)} e_u^2$

= {p(p +q /2 + (q / 2 – 1))^2} + {p(p +q /2 + (q / 2 – 1))^2} + {p(p +q /2 + (q / 2 – 2))^2} + {p(p +q /2 + (q / 2 – 2))^2}…+ {p(p +q /2)^2} + {p(p +q /2)^2} + {p(p +q /2 +1))^2} +…+ {p(p +q /2 + (q / 2 – 2))^2} + {p(p +q /2 + (q / 2 – 2))^2} + {p(p +q /2 + (q / 2 – 1))^2} + {p(p +q /2 + (q / 2 – 1))^2}

= 2p(p + q – 1)^2+ 2p(p + q – 2)^2 +…+ 2p(p + q / 2)^2 + 2p(p + q/2 + 1)^2 + …+ 2p(p +q – 2))^2+ 2p(p +q – 1))^2

$$= 2\sum_{k=0}^{q/2-1} 2p(p+q/2+k)^2$$

Theorem 3.3: Zagreb eccentric index of G = TUZC$_6$(p, q) is $E_2(G) = 2\sum_{k=0}^{q/2-1} 2p(p+q/2+k)^2 + 2\sum_{k=1}^{q/2-1} p((p+q/2+k)(p+q/2+(k-1))) + p(p+q/2)^2$

Proof: $E_2(G) = \sum_{uv\in E(G)} e_u e_v$

= {2p((p +q /2 + (q / 2 – 1) (p +q /2 + (q / 2 – 1))} + {p((p +q /2 + (q / 2 – 1) (p +q /2 + (q / 2 – 2))} + {2p((p +q /2 + (q / 2 – 2) (p +q /2 + (q / 2 – 2))} +… +{2p(p +q /2) (p +q /2)}+ {p(p +q /2) (p +q /2)} +{2p(p +q /2) (p +q /2)} + {p(p +q /2) (p +q /2 +1)} +…+ {2p((p +q /2 + (q / 2 – 2) (p +q /2 + (q / 2 – 2))} +{ p((p +q /2 + (q / 2 – 2) (p +q /2 + (q / 2 – 1))} +
{2p((p + q /2 + (q / 2 – 1) (p +q /2 + (q / 2 – 1))}

= 2p(p + q –1)^2 + p(p + q –1) (p + q –2) + 2p(p + q –2)^2 + …+ p(p + q/2)^2 + 2p(p + q/2)^2 + p(p +q /2) (p +q /2 +1) +…+ 2p(p + q –2)^2 + p (p + q –2) (p + q –1) + 2p(p + q –1)^2

$$= 2\sum_{k=0}^{q/2-1} 2p(p+q/2+k)^2 + 2\sum_{k=1}^{q/2-1} p((p+q/2+k)(p+q/2+(k-1))) + p(p+q/2)^2$$

Theorem 3.4: Eccentricity connectivity index of G = TUZC$_6$(p, q) is $\varepsilon^c(G) = 2\sum_{k=0}^{q/2-2} 6p(p+q/2+k) + 10p(p+q-1)$

Proof: $\varepsilon^c(G) = \sum_{u\in V(G)} d_u e_u$

= {p2(p +q /2 +(q / 2 – 1))} + {p3(p +q /2 + (q / 2 – 1))}+ {p3(p +q /2 +(q / 2 – 2))} + {p3(p +q /2 + (q / 2 – 2))} +... + {p3(p +q /2)} + {p3(p +q /2)}+ {p3(p +q /2)} + {p3(p +q /2)}+ …+ {p3(p +q /2 + (q / 2 – 2))} + {p3(p +q /2 + (q / 2 – 2))}+ {p3(p +q /2 + (q / 2 – 1))} + {p2(p +q /2 + (q / 2 – 1))}

= 5p(p + q – 1) + 6p(p + q – 2) +...+ 6p(p + q / 2) + 6p(p + q/2) +…+ 6p(p +q – 2) + 5p(p +q – 1)

$$= 2\sum_{k=0}^{q/2-2} 6p(p+q/2+k) + 10p(p+q-1)$$

Theorem 3.5: Multiplicative Zagreb eccentric index of G = $TUZC_6(p, q)$ is $\Pi E_1(G) = \prod_{k=0}^{q/2-1}\left(p^2(p+q/2+k)^4\right)^2$

Proof: $\Pi E_1(G) = \prod_{u\in V(G)} e_u^2$

$= \{(p(p+q/2+(q/2-1))^2)\}\{(p(p+q/2+(q/2-1))^2)\}\{(p(p+q/2+(q/2-2))^2)\}\{(p(p+q/2+(q/2-2))^2)\} \ldots \{(p(p+q/2)^2)\}\{(p(p+q/2)^2)\}\{(p(p+q/2)^2)\}\{(p(p+q/2)^2)\} \ldots \{(p(p+q/2+(q/2-2))^2)\}\{(p(p+q/2+(q/2-2))^2)\}\{(p(p+q/2+(q/2-1))^2)\}\{(p(p+q/2+(q/2-1))^2)\}$

$= (p^2(p+q-1)^4)(p^2(p+q-2)^4) \ldots (p^2(p+q/2)^4)(p^2(p+q/2)^4) \ldots (p^2(p+q-2))^4)(p^2(p+q-1))^4)$

$= \prod_{k=0}^{q/2-1}\left(p^2(p+q/2+k)^4\right)^2$

Theorem 3.6: Multiplicative Zagreb eccentric index of G = $TUZC_6(p, q)$ is $\Pi E_2(G) = \prod_{k=0}^{q/2-1}\left(2p(p+q/2+k)^2\right)^2 \prod_{k=1}^{q/2-1}\left(p((p+q/2+k)(p+q/2+(k-1)))\right)^2 \times p(p+q/2)^2$

Proof: $\Pi E_2(G) = \prod_{uv\in E(G)} e_u e_v$

$= (2p((p+q/2+(q/2-1)(p+q/2+(q/2-1)))(p((p+q/2+(q/2-1)(p+q/2+(q/2-2)))$

$(2p((p+q/2+(q/2-2)(p+q/2+(q/2-2))) \ldots (2p(p+q/2)(p+q/2))(p(p+q/2)(p+q/2)) \ldots (2p((p+q/2+(q/2-2)(p+q/2+(q/2-2)))(p((p+q/2+(q/2-2)(p+q/2+(q/2-1)))(2p((p+q/2+(q/2-1)(p+q/2+(q/2-1)))$

$= (2p(p+q-1)^2)(p(p+q-1)(p+q-2))(2p(p+q-2)^2) \ldots (2p(p+q/2)^2)(p(p+q/2)(p+q/2)) \ldots (2p(p+q-2)^2)(p(p+q-2)(p+q-1))(2p(p+q-1)^2)$

$= \prod_{k=0}^{q/2-1}\left(2p(p+q/2+k)^2\right)^2 \prod_{k=1}^{q/2-1}\left(p((p+q/2+k)(p+q/2+(k-1)))\right)^2 \times p(p+q/2)^2$

4.Conclusion

In this paper we computed Zagreb eccentric indices, Eccentricity connectivity index, Multiplicative Zagreb eccentric indices, Harmonic eccentricity index of armchair polyhex nanotubes and zigzag polyhex nanotubes.

5.References

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