EFFECT OF DUST PARTICLES ON UNSTEADY OSCILLATORY MHD FLOW OF THE FLUID THROUGH A PERMEABLE BOUNDARY OF THE CHANNEL

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Abstract

This paper is concerned with the effect of slip condition and thermal radiation on oscillatory flow of a conducting optically thin dusty fluid through a permeable boundary of the channel and the governing equations are solved analytically .The influence of various flow parameters such as Darcy number (Da) ,Grashof number (Gr), radiation parameter(N), Hartmann number (H) Prandtl number(Pr) and Reynolds number(Re) on velocity and skin friction of the fluid and dust particles and the Nusselt number across the channel are obtained and their physiological effects are discussed graphically.

Keywords: MHD, Permeable boundary, Dusty fluid, Thermal radiation , Oscillatory flow.

1. INTRODUCTION

Fluid flow with the influence of magnetic field and heat transfer has its importance in many applications like magneto-hydrodynamics accelerations, pumps , generators and this type of fluids has application in geothermal energy extraction, nuclear reactor, plasma studies and the boundary layer control in the field of aerodynamics. P.G.Saffman (1962) studied the stability of the laminar flow of a dusty gas and he proposed the equations of motion for the binary mixture of the fluid and the dust particles. The wave structure in oscillatory coquette flow of a dusty gas are studied by P.K.Kulshretha and P.Pure (1981).The effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field are investigated by O.D.Mankinde and A.Ogulu (2008). Prakash et.al.(2011) investigated the influence of radiative heat transfer on the MHD free convective flow of a viscoelastic dusty gas in a porous medium and the motion is induced by a semi-infinite flat plate.

The studies of flow and heat transfer on dusty fluids through a channel with permeable boundary are useful in the engineering fields and in many industrial area for improving the design and operation of the devices. The flow of dusty fluids has important applications in the fields of purification of crude oil, polymer technology, fluid droplet sprays, centrifugal separation of matter from fluid, petroleum industry, fluidization and combustion. Unsteady flow of a conducting dusty fluid through a rectangular channel with time dependent pressure gradient is studied by K.K.Singh (1976).V.R. Prasad and N.C.P. Ramacharyulu (1979) studied the unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient. A.K.Ghosh and D.K.Mitra (1984) studied the Flow of a dusty fluid through horizontal pipes. O.D.Makinde and T.Chinyoka (2010) studied MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition. Gireesha et.al.(2010)investigated the effect of heat transfer on unsteady flow of a dusty fluid throuugh a rectangle channel under the influence of uniform magnetic field and pulsatile pressure gradient. OM Prakash et.al.(2014) studied the effect of heat transfer on MHD oscillatory dusty fluid flow in a channel filled with a porous medium. In this paper , we studied the effect of slip condition and thermal radiation on oscillatory flow of a conducting optically thin dusty fluid through a permeable boundary and the problem is formulated, analyzed and solved analytically.The influences of various parameters in the dusty fluid flow through a channel are discussed quantitatively and their physical effects are presented through the graphs.

2.MATHEMATICAL FORMULATION

Consider the flow of the dusty fluid in a channel with permeable boundary under the influence of radiative heat transfer .The homogenous magnetic field is applied externally to the dusty fluid and it is assumed to have small electrical conductivity. Cartesian coordinate system (x,y) are taken, x lies along the centre of the channel and y is the distance measured in the normal direction.

Assumptions

1. The dust particles are assumed to be spherical, solid, non conducting and equal in size.

2. It is uniformly distributed in the flow region.Their number density No is constant and the temperature between the particles is uniform throughout the motion of the fluid .

3. The chemical reaction and the interactions between the particles have not been considered.

4. The Magnetic Reynolds number is taken to be very small so that induced magnetic field is negligible and Hall effects have been neglected.so the flow region has uniform applied magnetic field and uniform temperature .

5. The dust particles are transported within the fluid and uniformly distributed such that the continuity equation is satisfied.

Assuming a Boussinesq incompressible fluid model, the equations governing the motion and energy balance are as follows:

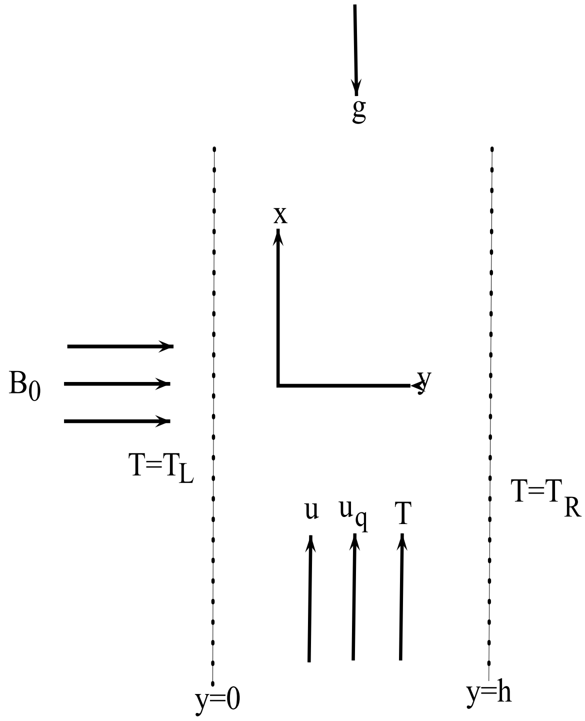

Figure 1: Geometry of the problem

$$\frac{\partial u}{\partial t} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{N_0 K_0}{\rho}(u_q - u) - \frac{\sigma_e B_0^2}{\rho}u + g\beta(T - T_0) \tag{1}$$

$$\frac{\partial u_q}{\partial t} = k_0(u - u_q) \tag{2}$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y} \tag{3}$$

The initial and boundary conditions are given as

$$\left.\begin{aligned}
&u = u_p \text{ and } \frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{Da}}(u_B - u_p) \ \ at \ \ y = h \\
&\frac{\partial u}{\partial y} = 0 \ \ at \ \ y = 0 \\
&T(y,0) = T_F \\
&T(a,t) = T_R = \ T_L + (T_F - T_L)e^{i\omega t} \\
&T(0,t) = T_L
\end{aligned}\right\} \tag{4}$$

where u is the velocity of the fluid ,u_q is the velocity of the dust particles in the x-direction, t is the time, ω is the frequency of the oscillation, T is the fluid temperature, T_f is the wall temperature, p is the fluid pressure, g is the gravitational force, T_L is the left wall temperature, T_R is the right wall temperature, q is the radiative heat flux, β is the co-efficient of volume expansion due to temperature, $k_0 = (6x\rho\upsilon D)$, (D is the average radius of dust particles) is the stokes constant pressure, k is the thermal conductivity, ρ is the fluid density, C_p is the specific heat at constant pressure, σ_e is the conductivity of the fluid. υ is the kinematic viscosity. B_0 is the electromagnetic induction. μ_e is the Magnetic Permeability and H_0 is the intensity of Magnetic field.

The fluid is assumed to be optically this with a relatively low density and the radiative heat flux is given by Cogly et al(1968).

$$\frac{\partial q}{\partial y} = 4\alpha^2(T_0 - T) \tag{5}$$

where α is the mean radiation absorption co-efficient.

The following dimensionless variables and parameters are introduced.

$$\bar{x} = \frac{x}{h},\ \ \bar{y} = \frac{y}{h},\ \ \bar{u} = \frac{u}{U},\ \ \theta = T - T_0,\ \ \bar{t} = \frac{tU}{h},\ \ M = \frac{\upsilon}{k_0 h^2},\ \ l = \frac{N_0 K_0 h^2}{\rho\upsilon},\ \ R_e = \frac{uh}{\upsilon},$$
$$pr = \frac{\upsilon\rho C_p}{k},\ \ N^2 = \frac{4\alpha^2 a^2}{k},\ \ \bar{u_p} = \frac{u_p}{U},\ \ Gr = \frac{g\beta(T_F - T_L)a^2}{\upsilon U},\ \ u_P = \frac{\overline{u_P}}{a},\ \ u_B = \frac{\overline{u_B}}{a},\ \ \bar{p} = \frac{hp}{\upsilon\rho\upsilon},$$
$$H^2 = \frac{h^2\sigma_e B_0^2}{\upsilon\rho},\ \ S^2 = \frac{1}{D_a} \tag{6}$$

where U is the flow mean velocity.

Substituting the dimensionless variables (6) in to equation (1) to equation (3), we get (by omitting the bars)

$$R_e \frac{\partial u}{\partial t} = \frac{-\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (H^2 + l)u + lu_q + Gr\theta \tag{7}$$

$$R_e\ M\ \frac{\partial u_q}{\partial t} = (u - u_q) \tag{8}$$

$$R_e\ pr\ \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \tag{9}$$

The corresponding initial and boundary conditions are given as

$$\left.\begin{aligned} & u = u_p \text{ and } \frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{Da}}(u_B - u_p) \ \ at \ \ y = 1 \\ & \frac{\partial u}{\partial y} = 0 \ \ at \ \ y = 0 \\ & \theta(y, 0) = 1 \\ & \theta(1, t) = e^{i\omega t} \\ & \theta(0, t) = 0 \end{aligned}\right\} \tag{10}$$

u_p is the velocity in the permeable boundary, u_B is the slip velocity, Da is the Darcy number, Gr is the Grashof number, H is the Hartmann number, l is the particle concentration parameter, M is the particle mass number, N is the radiation parameter, Re is the flow Reynolds number and pr is the prandtl number.

3. SOLUTION OF THE PROBLEM

In order to solve equations (7), (8) and (9) subject to the boundary condition equation

(10), we assume the solutions of the form

$$\frac{-\partial p}{\partial x} = Ae^{i\omega t} \tag{11}$$

$$u(y,t) = u_0(y)e^{i\omega t} \tag{12}$$

$$u_q(y,t) = u_{q_0}(y)e^{i\omega t} \tag{13}$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t} \tag{14}$$

where A is constant oscillation amplitude for pressure gradient.

$u(y,t)$, $u_q(y,t)$ and $\theta(y,t)$ are to be determined.

Substituting the equations (11) to (14) into equations (7) to (9), comparing the harmonic and non-harmonic terms, we obtain

$$\frac{\partial^2 u_0}{\partial y^2} - C_1^2 u_0(y) = -A - Gr\theta_0(y) \tag{15}$$

$$u_{q_0} = \frac{u_0}{1 + i\omega R_e M} \tag{16}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + C_2^2 \theta_0 = 0 \tag{17}$$

Now, the boundary conditions becomes

$$\left.\begin{array}{l} u_0 = u_{p_0} \text{ and } \dfrac{\partial u_0}{\partial y} = \dfrac{\alpha}{\sqrt{Da}}(u_{B_0} - u_{p_0}) \ \ at \ \ y = 1 \\ \dfrac{\partial u_0}{\partial y} = 0 \ \ at \ \ y = 0 \\ \theta_0 = 0 \ \ at \ \ y = 0 \\ \theta_0 = 1 \ \ at \ \ y = 1 \end{array}\right\} \tag{18}$$

Solving the equations (15) to (17), using the boundary conditions equation (18), we obtain the velocity profile for the fluid and dust particles, and the temperature for the fluid.

The temperature profile for the fluid is

$$\theta_0 = \frac{\sin(C_2 y)}{\sin C_2}$$
$$\theta = \left[\frac{\sin(C_2 y)}{\sin C_2}\right] e^{i\omega t} \tag{19}$$

The solution for the velocity of the fluid is

$$u(y,t) = \left[A_3 e^{C_1 y} + A_4 e^{-C_1 y} + \frac{\lambda}{C_1^2} + \frac{Gr}{C_1^2 + C_2^2}\left[\frac{\sin(C_2 y)}{\sin C_2}\right]\right] e^{i\omega t} \tag{20}$$

The solution for the velocity of the dust particles

$$u_q(y,t) = \left[\frac{u}{1 + i\omega R_e M}\right] e^{i\omega t} \tag{21}$$

Using the boundary conditions equations (18), the values of the coefficients A_1, A_2, A_3 and A_4 are obtained.

Using the equation (20), the skin friction for the fluid is given as

$$C_f = \left[\frac{\partial u}{\partial y}\right]_{y=1}$$
$$C_f = \left[A_3 C_1 e^{C_1} - A_4 C_1 e^{-C_1} + \frac{Gr}{C_1^2 + C_2^2} C_2 \cot(C_2)\right] e^{i\omega t} \tag{22}$$

Using the equation(21) ,We obtain the skin friction for the dust particles as

$$C_{fp} = \left[\frac{\partial u_q}{\partial y}\right]_{y=1}$$
$$C_{fp} = \frac{e^{i\omega t}}{1 + i\omega R_e M}\left[A_3 C_1 e^{C_1} - A_4 C_1 e^{-C_1} + \frac{Gr}{C_1^2 + C_2^2} C_2 \cot(C_2)\right] \tag{23}$$

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Using the equation (19),we obtain the rate of heat transfer across the channel's wall

$$N_u = -\left[\frac{\partial\theta}{\partial y}\right]_{y=1}$$

$$N_u = -[c_2 cot(c_2)]e^{i\omega t} \tag{24}$$

4.RESULTS AND DISCUSSION

The objective of this paper is to study the effect of slip condition and thermal radiation on oscillatory flow of a optically thin conducting dusty fluid through a permeable boundary and the governing equations are solved analytically .The influence of various flow parameters such as Darcy number (Da),Grashof number (Gr), radiation parameter(N), Hartmann number (H) Prandtl number(Pr) and Reynolds number(Re) on velocity and skin friction of the dusty fluid and dusty particle and the Nusselt number across the channel are obtained and their physiological effects are discussed graphically. In order to have an estimate of the physical effects of the various parameters involved in the flow analysis, software MATLAB 2013a is used to depicted the graphs .We have used the following parameter values Da=2,3,4,5;Gr=3,4,5,6;H=1,2,3;Pe=3,4,5;N=1,2,3,4,5; M=2,3,4;Re=1,2,3,4,5.The prandtl number(Pr) is fixed as 0.71 for the velocity and the temperature profiles which is corresponds to the atmospheric environment (air) at 20^0C. Figure 1 , shows that an increase in the Darcy number (Da),increases the fluid velocity. Figure 2 , shows the fluid velocity for different values of Grashof number(Gr). It is clear from the figure that an increase in the Grashof number(Gr) increases the fluid velocity because of the buoyancy force. Moreover, the maximum velocity of the dusty fluid occurs within the centerline region of the channel(i.e.y=0.5). Figure 3 shows that an increase in the Hartmann number(H) increases the velocity of the dusty fluid.It is observed that the maximum flow occurs in the presence of magnetic field. Figure 4 , shows the velocity of the fluid for different values of particle mass parameter (M). It is clear from the figure that an increase in the particle mass parameter (M) , increases the fluid velocity.Figuren5

, shows the fluid velocity for different values of radiation parameter(N). It is clear from the figure that an increase in the radiation parameter(N) decreases the velocity of the fluid.

Figure 6, shows the velocity of the dust particles for different values of Grashof number(Gr). This shows that an increase in the Grashof number(Gr)increases the velocity of the dust particles (u_q). Figure 7, shows that an increase in the Hartmann number(H) decreases the velocity of the dust particles. This is due to the rise in the magnetic field intensity.The conducting fluid particles with transversely imposed magnetic field produces the Lorentz force which acts as a resistance to the flow,so there is an decrease in the velocity of the dusty particle while increasing the Hartmann number. Figure 8 , shows that an increase in the particle mass parameter (M) decreases the velocity of the dust particles.

Figure 9, shows that an increase in the radiation parameter(N) decreases the temperature (θ) of the fluid.Figure 10, shows that an increase in the Reynolds number(Re) decreases the temperature (θ) of the fluid. Figure 11 , shows that an increase in the particle mass parameter (M) decreases the skin friction of the fluid. particle mass parameter (M). Figure 12 , shows that an increase in the Grashof number(Gr) increases the skin friction of the fluid. Figure 13 , shows that an increase in the Hartmann number(H) decreases the skin friction of the fluid.This is because of the reason that effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force called Lorentz force which is similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. Figure 14, shows that an increase in the radiation parameter(N) increases the skin friction of the fluid.

Figure 15 , shows that an increase in the Grashof number(Gr) increases the skin friction of the dust particles. Figure 16, shows the decreases of the Nusselt number(Nu) while increasing radiation parameter(N). Finally the Figure 17, shows the Nusselt number(Nu) decreases for the increasing values of the Reynolds number(Re).

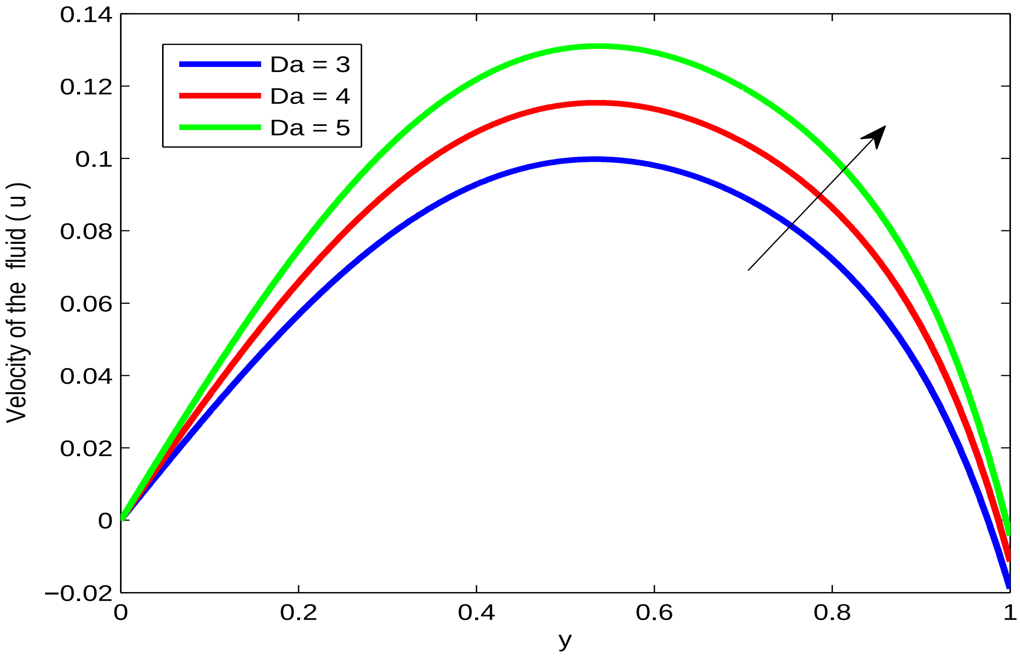

Figure 2: Variation of velocity of the fluid (u) for different values of Darcy number (Da) for fixed $Re = 3, l = 1.5, N = 2, Pr = 0.71, H = 2, Gr = 4, M = 2, \omega = 1$

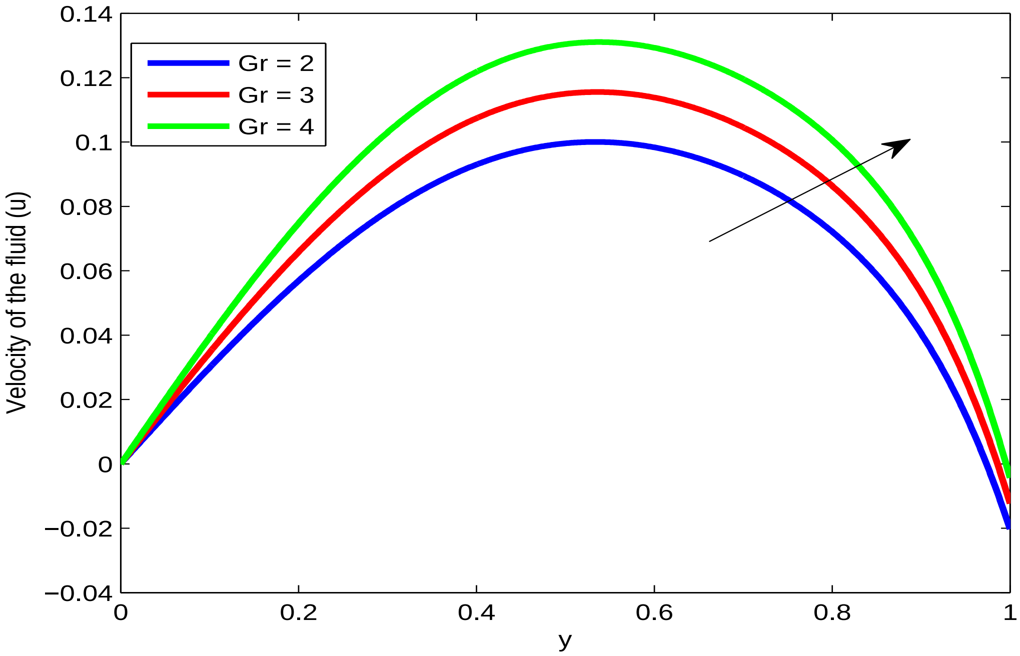

Figure 3: Variation of velocity of the fluid (u) for different values of Grashof number(Gr) for fixed $Re = 3, l = 1.5, N = 2, Pr = 0.71, H = 2, Da = 3, M = 2, \omega = 1$

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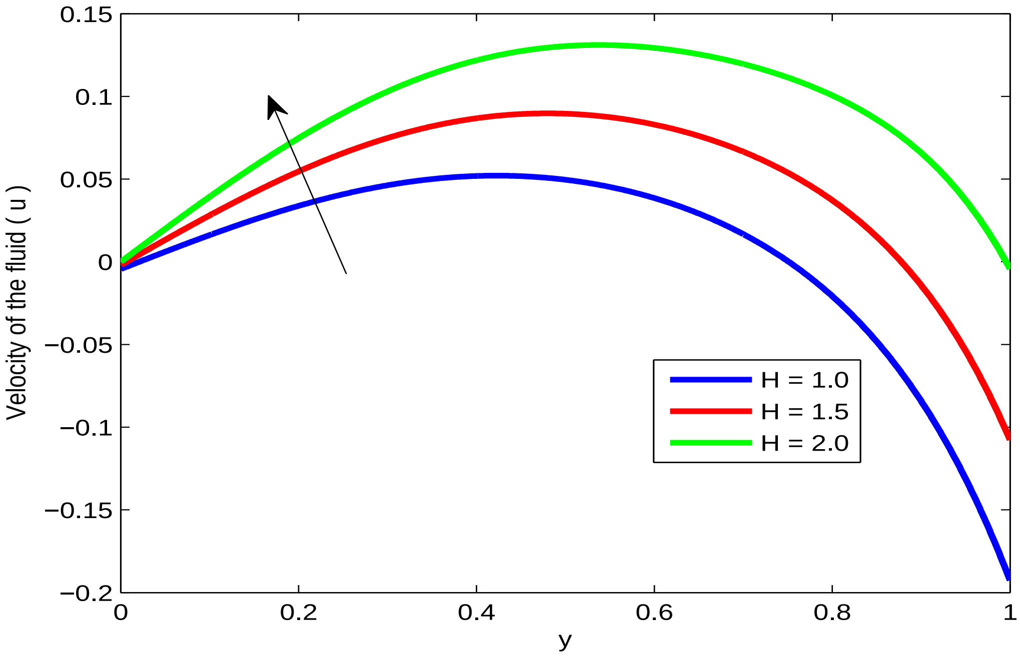

Figure 4: Variation of velocity of the fluid (u) for different values of Hartmann number(H) for fixed $Re = 3, l = 1.5, N = 2, Pr = 0.71, Gr = 4, Da = 3, M = 2, \omega = 1$

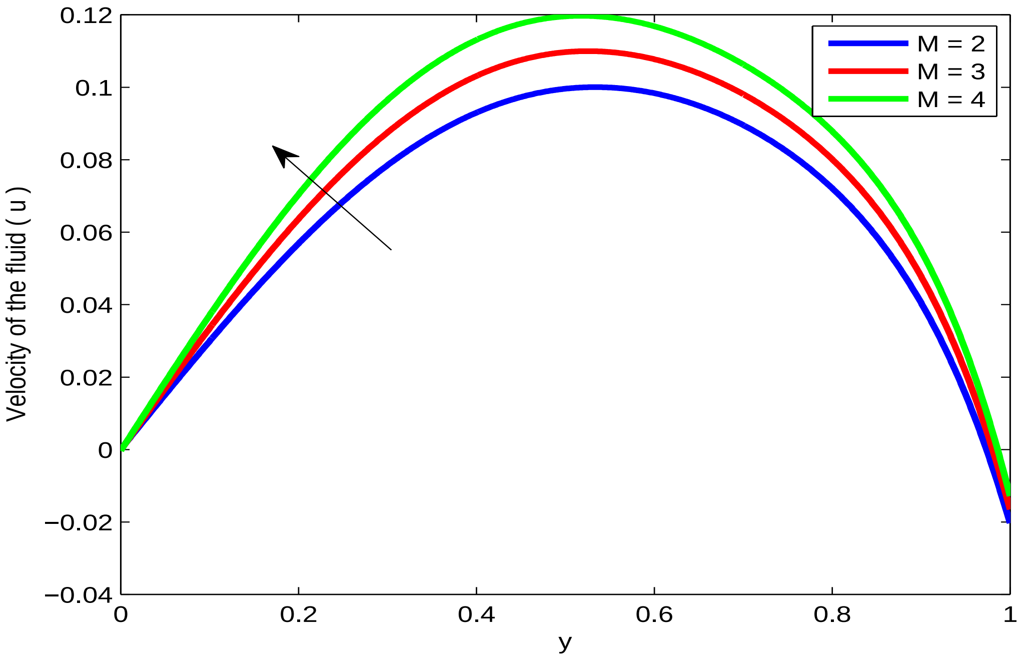

Figure 5: Variation of velocity of the fluid (u) for different values of the particle mass parameter $(M)$for fixed $Re = 3, l = 1.5, N = 2, Pr = 0.71, Gr = 4, Da = 3, H = 2.5, \omega = 1$

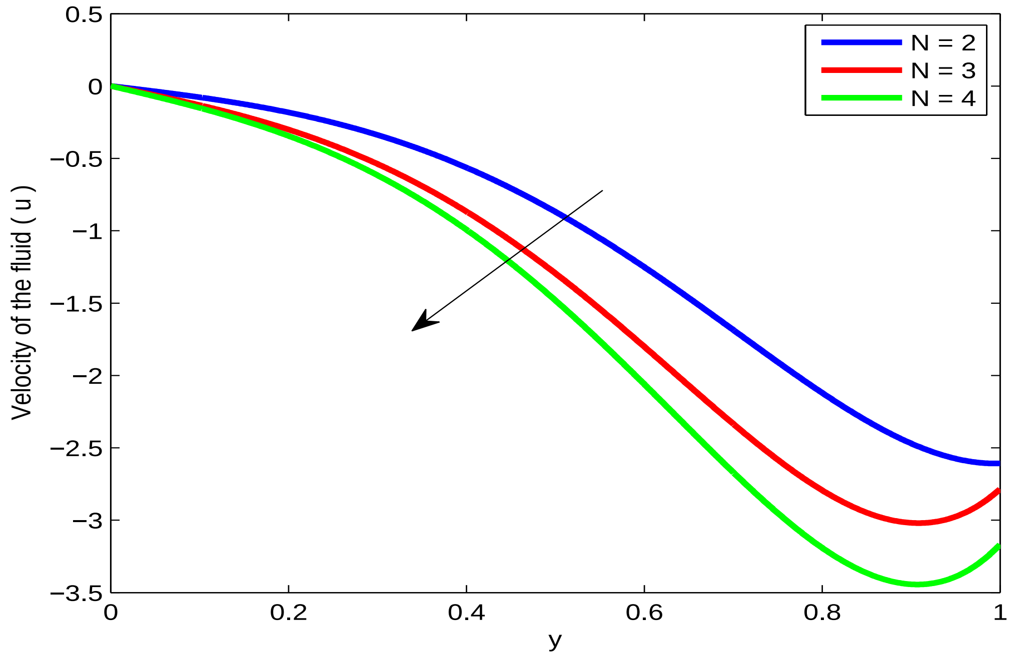

Figure 6: Variation of velocity of the fluid (u) for different values of radiation parameter$(N)$for fixed $Re = 3, l = 1.5, Gr = 2, Pr = 0.71, Da = 3, H = 2, M = 2, \omega = 1$

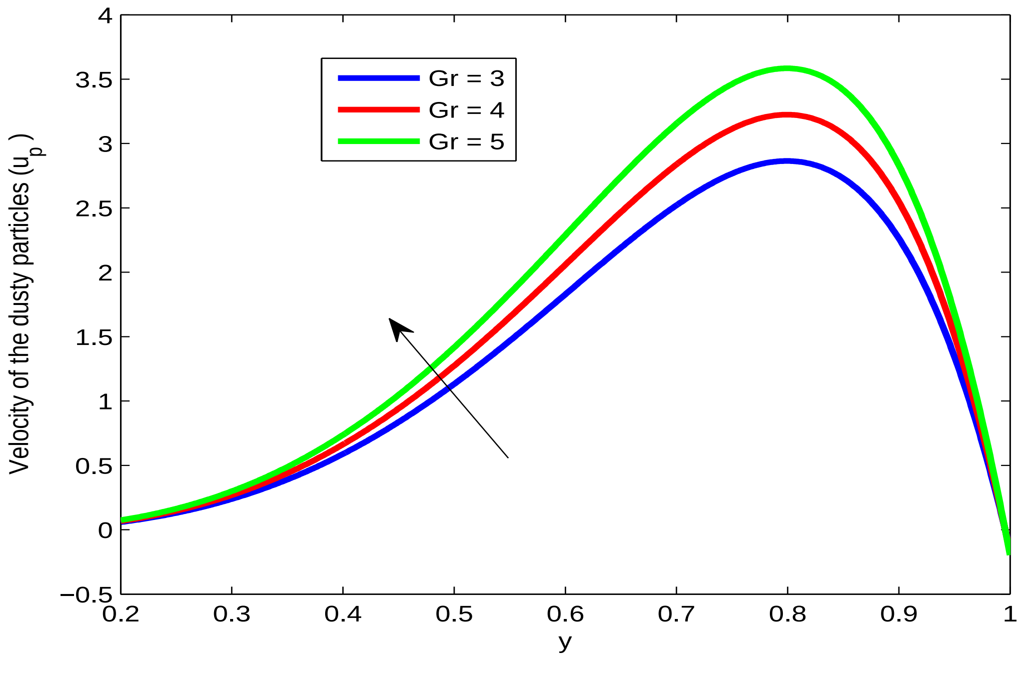

Figure 7: Variation of velocity of the dust particles (u_q) for different values of Grashof number$(Gr)$for fixed $Re = 2, l = 1.5, N = 2, Pr = 0.71, H = 2, Da = 4, M = 2, \omega = 1$

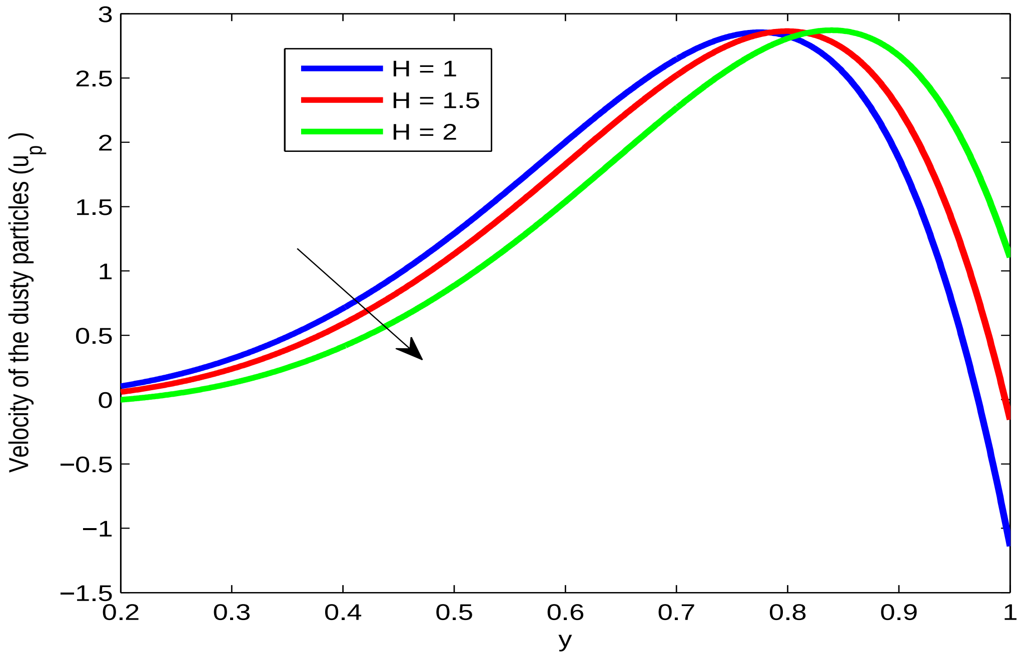

Figure 8: Variation of velocity of the dust particles (u_q) for different values of Hartmann number(H)for fixed $Re = 4, l = 1.5, N = 2, Pr = 0.71, Gr = 4, Da = 3, M = 2, \omega = 1$

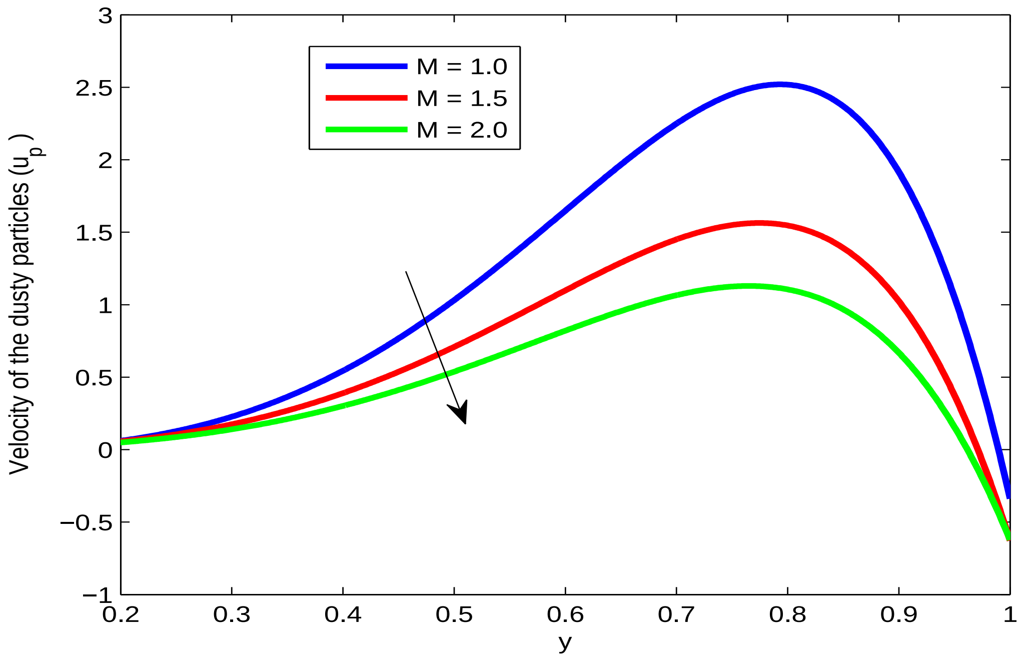

Figure 9: Variation of velocity of the dust particles (u_q) for different values of the particle mass parameter(M) for fixed $Re = 4, l = 1.5, N = 2, Pr = 0.71, Gr = 4, Da = 3, H = 2, \omega = 1$

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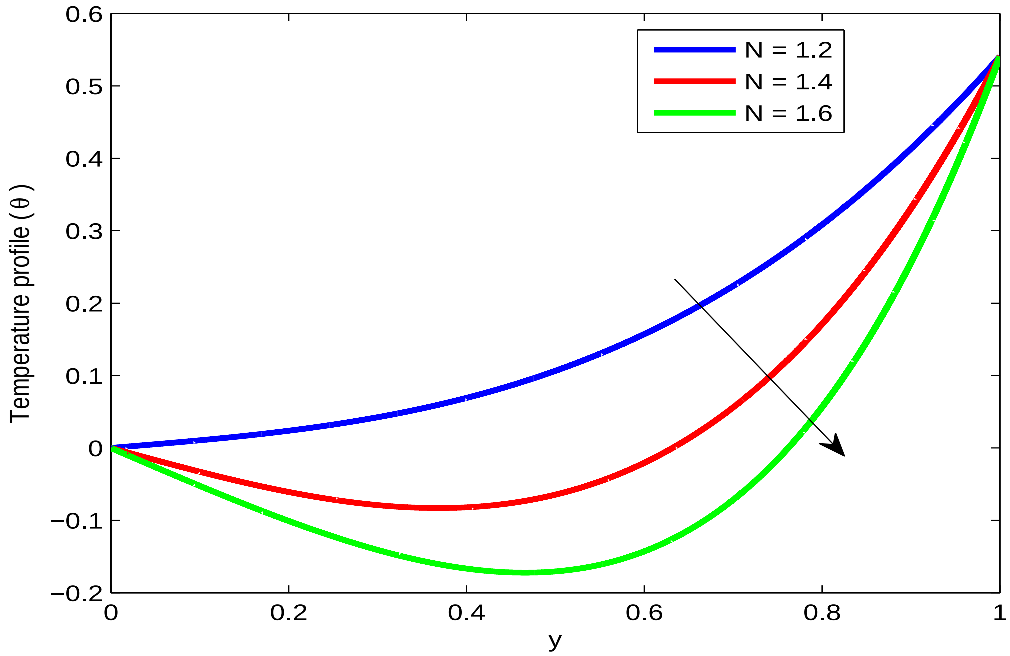

Figure 10: Variation of Temperature of the fluid (θ) for different values of radiation parameter$(N)$for fixed $Re = 3, t = 1, Pr = 0.71, \omega = 1$

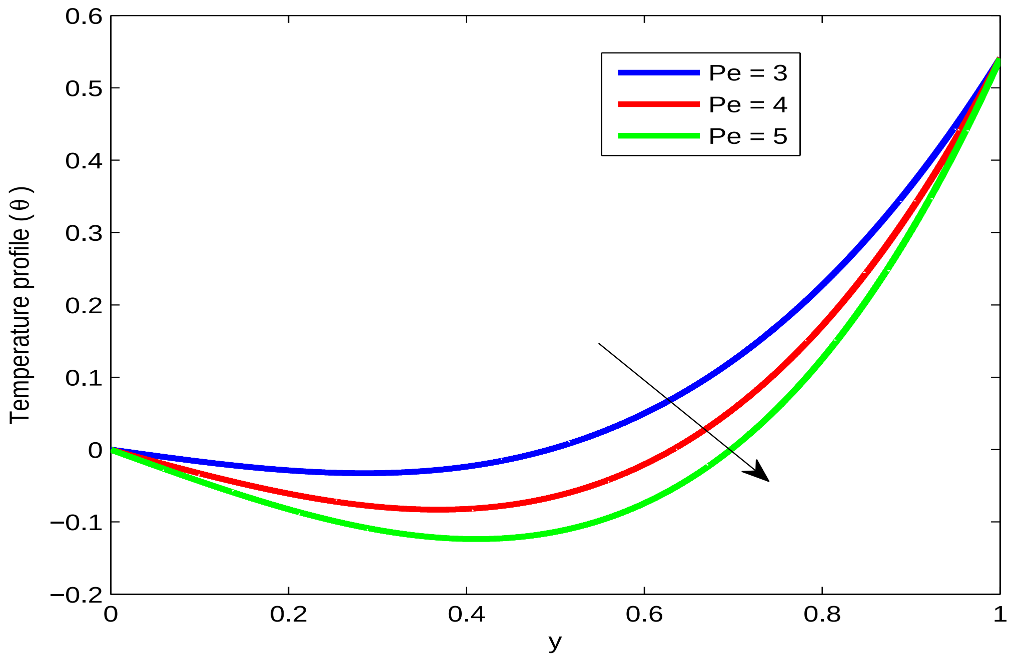

Figure 11: Variation of Temperature of the fluid (θ) for different values of Reynolds number $(Re)$for fixed $N = 1.5, t = 1, Pr = 0.71, \omega = 1$

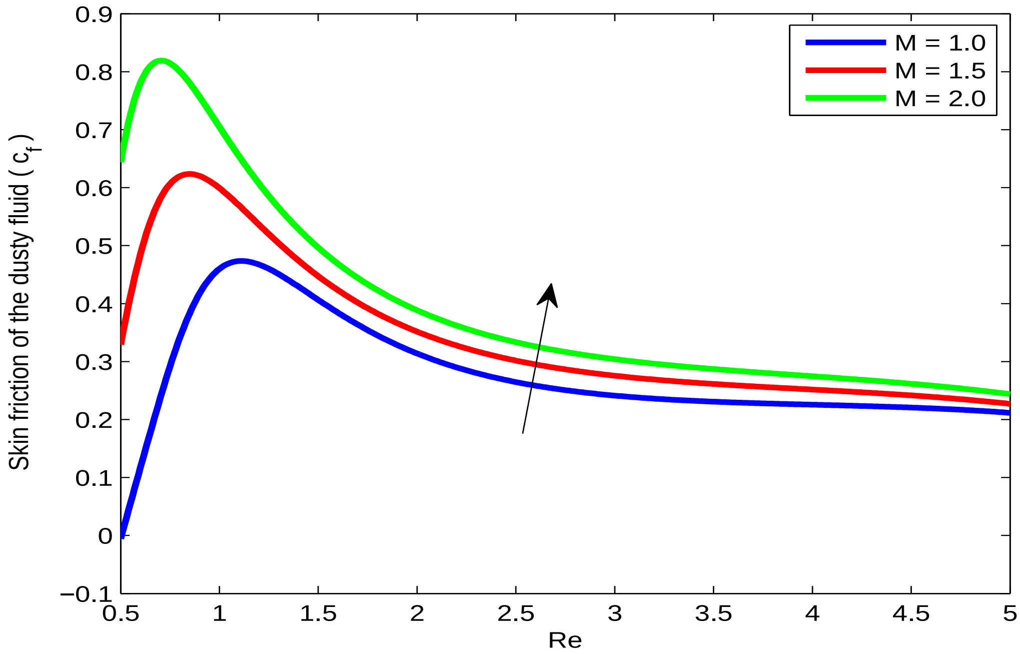

Figure 12: Skin friction of the fluid (C_f) for different values of the for different values of the particle mass parameter(M) for fixed $l = 1.5, N = 2, Pr = 0.71, Gr = 4, Da = 3, H = 2, \omega = 1$

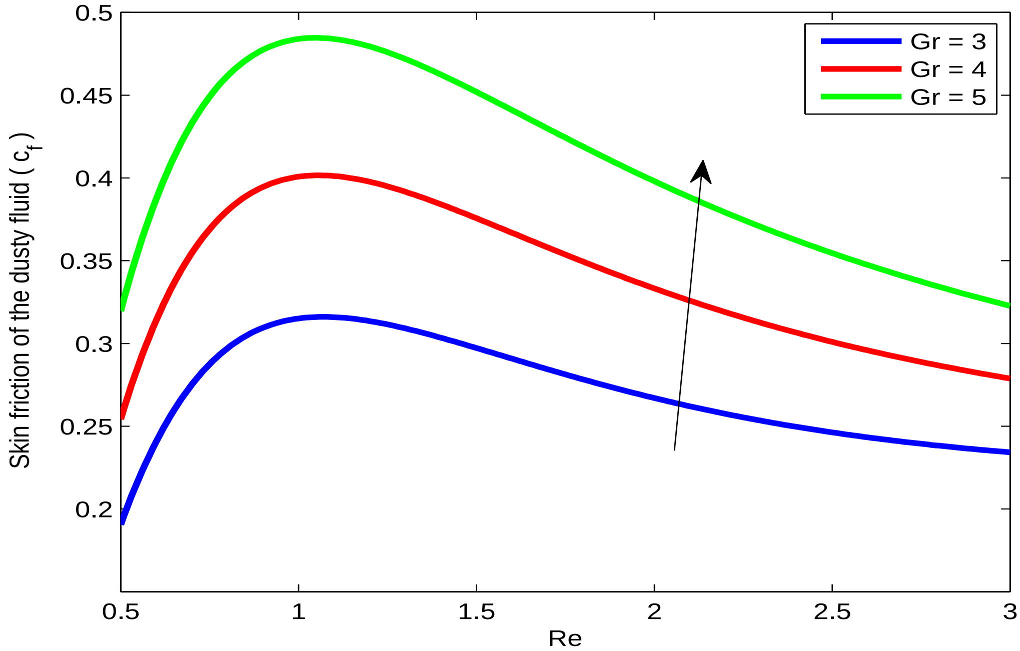

Figure 13: Skin friction of the fluid (C_f) for different values of the for different values of Grashof number (Gr)for fixed $l = 1.5, N = 2, Pr = 0.71, Da = 3, H = 2, \omega = 1$

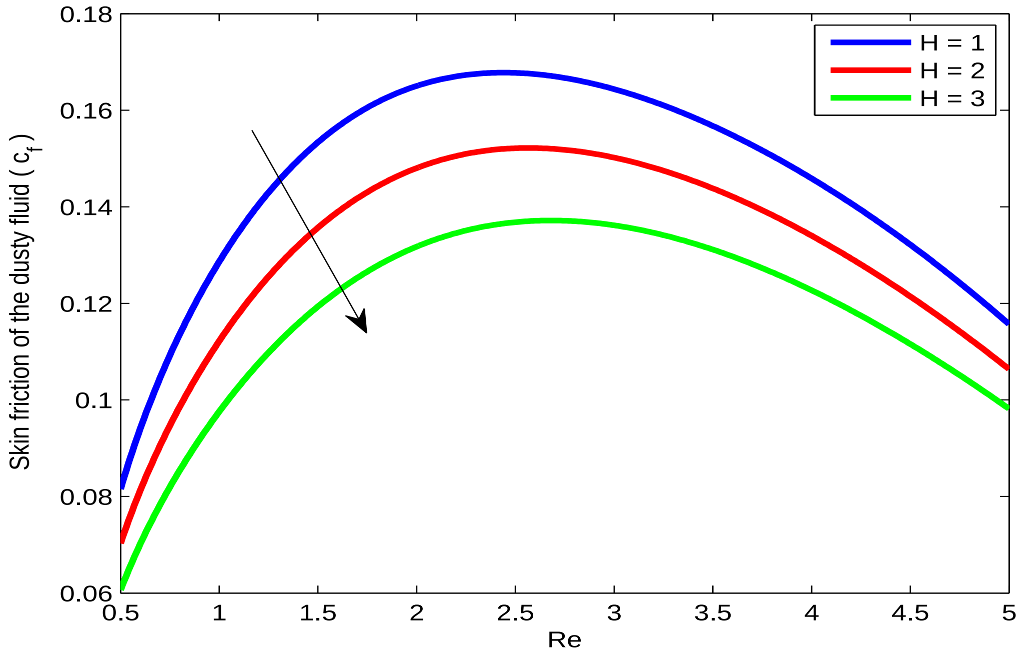

Figure 14: Skin friction of the fluid (C_f) for different values of the for different values of Hartmann number(H) for fixed $l = 1.5, N = 2, Pr = 0.71, Gr = 4, Da = 3, M = 2, \omega = 1$

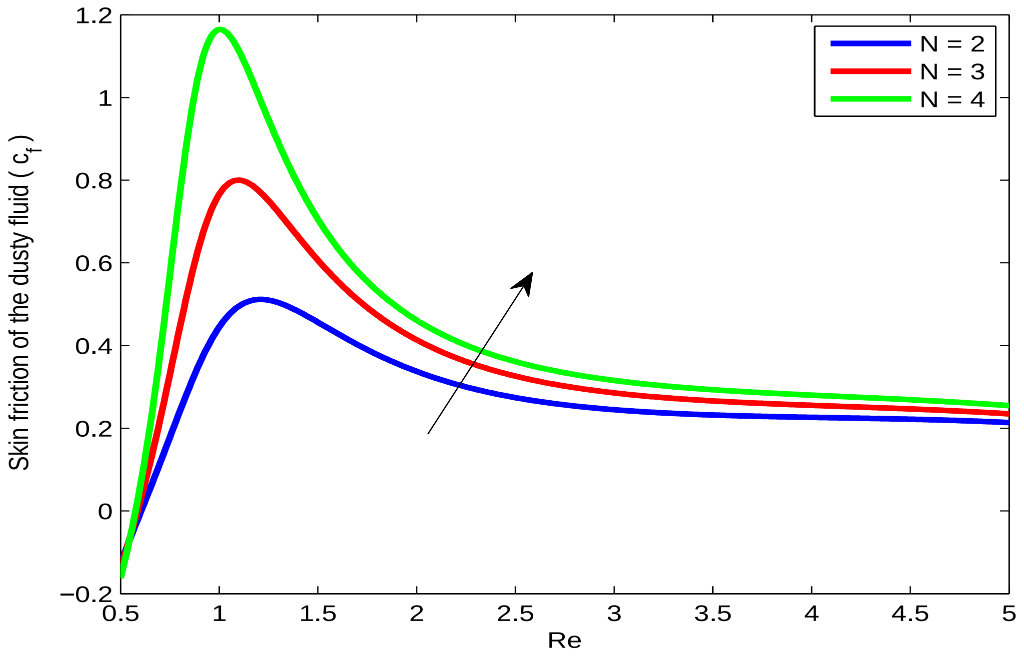

Figure 15: Skin friction of the fluid (C_f) for different values of the for different values of the radiation parameter(N) for fixed $l = 1.5, M = 1.5, Pr = 0.71, Gr = 4, Da = 3, H = 2, \omega = 1$

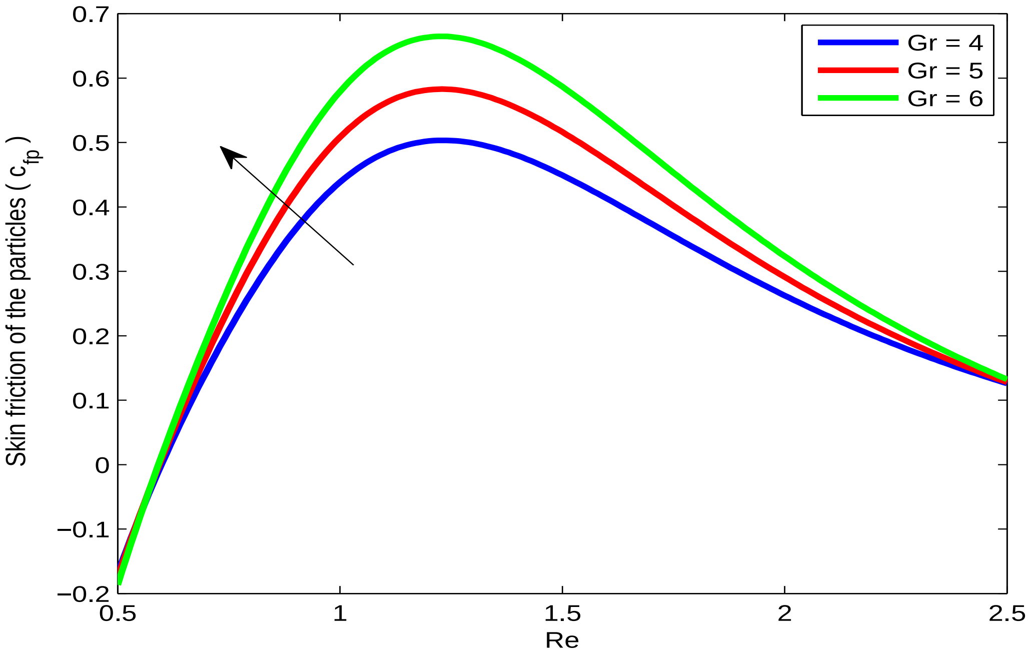

Figure 16: Skin friction of the dust particles (C_{fp}) for different values of Grashof number $(Gr)$for fixed $l = 1.5, M = 2.5, N = 2, Pr = 0.71, Da = 3, H = 2, \omega = 1$

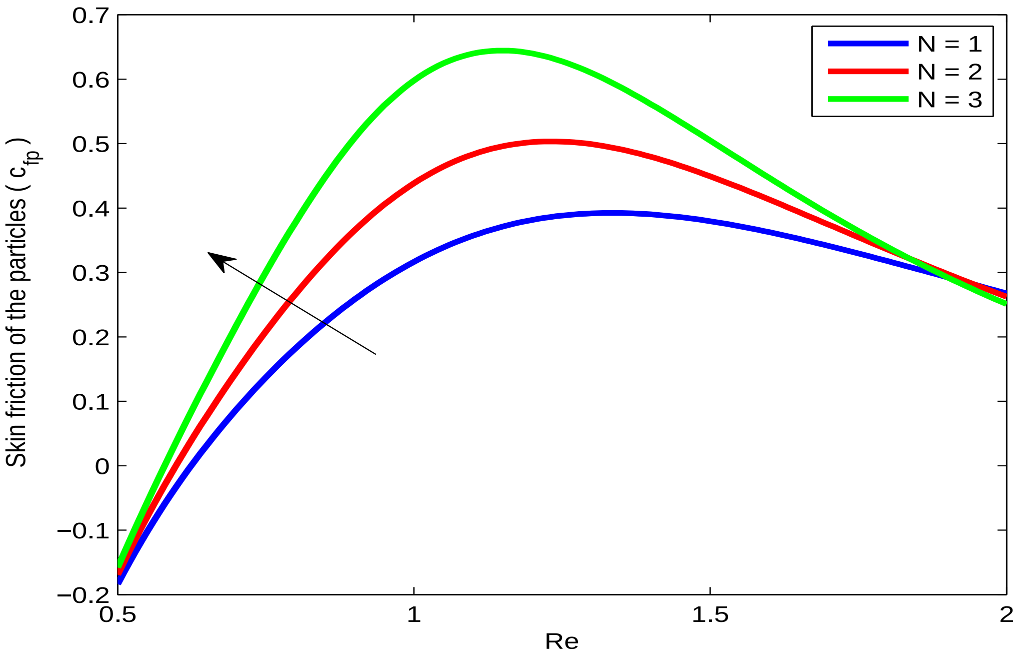

Figure 17: Skin friction of the dust particles (C_{fp}) for different values of the radiation parameter(N) for fixed $l = 1.5, M = 2, Pr = 0.71, Gr = 4, Da = 3, H = 2, \omega = 1$

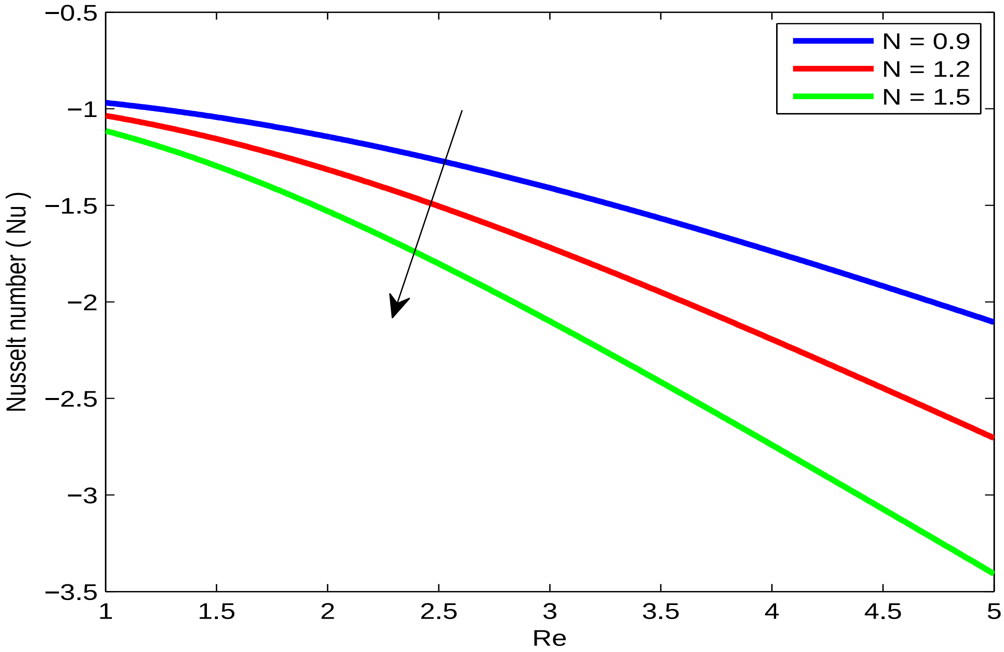

Figure 18: Variation of Nusselt number of the fluid(Nu) for different values of the radiation parameter(N) for fixed $Pr = 0.71, t = 1, \omega = 1$

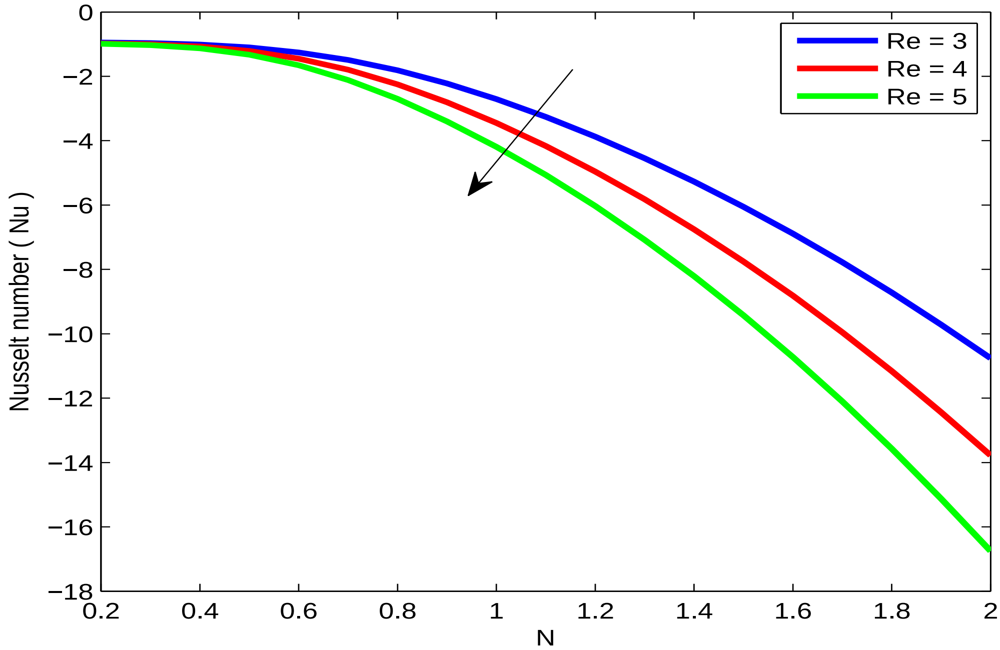

Figure 19: Variation of Nusselt number of the fluid(Nu) for different values of the Reynolds number (Re) for fixed $Pr = 0.71, t = 1, \omega = 1$

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Appendix

$$A_1 = 0$$

$$A_2 = \frac{1}{\sin(c2)}$$

$$A_3 = \frac{\frac{Gr}{c_1^2+c_2^2}\frac{c_2}{\sin(c_2)}\left[2c_1\sinh(c_1) + (e^{c_1} - \cot(c_2)) - \frac{\alpha}{\sqrt{Da}}[u_{B_0} - u_{p_0}]\right]}{2c_1\sinh(c_1)}$$

$$A_4 = \frac{\frac{Gr}{c_1^2+c_2^2}\left[\frac{e^{c_1}}{\sin(c_2)} - \cot(c_2)\right] - \frac{\alpha}{\sqrt{Da}}[u_{B_0} - u_{p_0}]}{2c_1\sinh(c_1)}$$

$$c_1^2 = \left[H^2 + i\omega Re + \frac{1}{(1+i\omega ReM)}\right]$$

$$c_2^2 = \sqrt{N^2 - i\omega RePr}$$

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