Design of IIR-Based Digital Differentiator-A Survey

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Abstract: *A differentiator is a signal processing device that determines the time derivative of a given signal for exercising velocity and acceleration estimation in radars, biomedical investigations, image processing. Finite impulse response digital differentiators provide stable and linear responses over a wide range of frequency compared to infinite impulse response digital differentiators, which are complex in design, despite they stand in need for the applications where the efficient signal processing is needed. To date, many conventional, as well as evolutionary optimization techniques have been employed by researchers to design IIR-based digital differentiators. Although conventional methods are productive for unimodal problems, they model a deficiency in multi-modal problems. On the contrary, natural selection and evolution-based optimization techniques favor diversity in choosing the optimal solution with lesser control parameters. However, there is always a scope of improvement in parameter optimization. This paper aims to cover the study and analysis of both the methods for designing digital differentiators and their respective comparison in terms of magnitude and phase response error metrics that have also been mentioned.*

Keywords: Digital Differentiator, Digital Integrator, IIR Filter, Conventional Techniques, Evolutionary Optimization Techniques, Magnitude Response Error.

1. Introduction

Digital filters, being an intrinsic part of digital signal processing, perform two essential functions of signal separation and signal restoration. A good digital filter gives the desired magnitude response and linear phase. Implementation of digital filters is possible in two ways: by convolution (also called Finite Impulse Response, FIR) and by recursion (also called Infinite Impulse Response, IIR). High selectivity of IIR filter at low frequency makes it more realizable than the FIR filter [1]. Unlike FIR filters, the transfer function of the IIR filter contains both poles and zeros, so exhibit more general structure and as a result, they can effectively approximate the desired response with that of the ideal filter response. Thus IIR filters outperform FIR filters having the same number of coefficients.

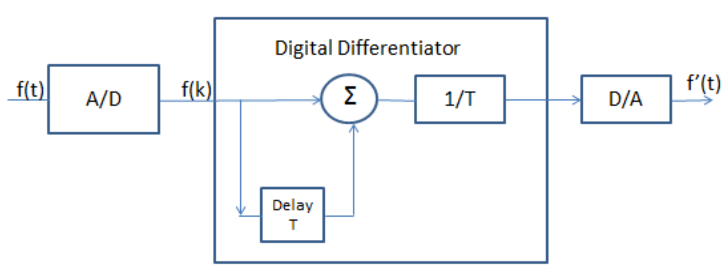

Figure 1. Block Diagram of Digital Differentiator

Differentiation is a powerful tool in signal processing for the determination and estimation of the incoming signal [2]. The functional block diagram of digital differentiator is shown in Figure 1. Digital differentiators are used to finding the time-derivation of the incoming signal used in the fields like biomedical engineering, control system, digital signal, image processing, and instrumentation [2]. Ideal differentiator has the frequency response given by equation 1.

$$H\left(e^{j\omega}\right) = j\omega\ ;\ |\omega| \leq \pi \quad (1)$$

Digital Differentiators have two types; FIR and IIR. FIR digital differentiators do not require much memory and they are stable over the wide frequency range and have a linear frequency response when compared to IIR digital differentiators [3]. Despite they are preferred in applications that require excellent signal processing.

Many conventional and evolutionary optimization approaches have been adopted by researchers to design digital differentiators. Conventionally, the inversion of the transfer function of integrators is employed after some modifications to design digital differentiators for which different Newton-Cotes integrators are first designed using interpolation techniques. But these methods can only efficiently optimize the unimodal search space and are unfaithful to solve multi-modal problems [4]. They demonstrate the following drawbacks in computing problems that involve multi-modal optimization [5].

i. As they are limited up to small search space.
ii. They show early convergence to a local optimal solution.
iii. They stand in need of continuous and differentiable error fitness function.
iv. If the number of variables gets increased, they show sensitivity to starting points.
v. They require a piecewise linear cost function when the linear programming method has to be applied.

To overcome the shortcomings of conventional methods; nature-inspired heuristic and meta-heuristic evolutionary optimization techniques are utilized that have been discussed in this literature.

A differentiator should have a wideband frequency response and minimum group delay for implementation in real-time applications [6]. Like any other digital filter, digital differentiators may be realized in FIR or IIR configuration. FIR-based designs of digital differentiators have perfect linear phase. Various techniques have been adopted to design digital differentiators of FIR type having linear phase characteristics for low frequency [7], mid-band frequency [8], high frequency [9], and wideband frequency applications [10]. However, they have long filter length and high filter order. Also, in most applications where the perfectly linear phase is not a big requirement, IIR differentiators prove to be more attractive than FIR differentiators [11]; for they can satisfy the given filter specifications with a much lower filter order and also having much smaller group delay.

Based on the parameters such as range and accuracy in magnitude response approximation of designed differentiators with ideal ones, phase response, relative error in magnitude and phase response, group delay, convergence rate, stagnation, unimodal and multi-modal nature of the problem, capability of searching large search space, various design methodologies have been proposed for the design of IIR-based digital differentiators in this literature review.

This review is organized as follows: Section 2 describes the literature review. This section provides a momentary explanation of the related work studies in the designing of IIR digital differentiators. In section 3, a discussion on the approaches used in selected papers is presented and section 4 demonstrates the conclusion.

2. Related Works

This section provides a review based on the design of digital IIR differentiator using different methods proposed by researchers. The survey is distributed in two parts: one; in which classical or conventional methods have been used and the other; in which evolutionary optimization algorithms have been applied. The first key area concerns the conventional approach that was used for the designing of analog differentiators by using integrators and then we move to the evolutionary optimization techniques that have been deployed to optimize the filter coefficients.

2.1 Design of IIR Digital Differentiator using Conventional Methods

Al-Alaoui [12], in 1992, derived a class of digital integrators from Tick Integrator having range and accuracy the same as the derived digital differentiators. Then transfer function of the resulting integrators was inverted after some modifications in pole-zero locations and compensating the magnitude for designing digital differentiators. In the continuation of this approach, Al-Alaoui [13], in 1993, then used the basic concept of interpolating the two well-known numerical integration techniques namely rectangular and trapezoidal rules at half the Nyquist frequency. As a result, the new integrator overlapped the range and accuracy of both the integrators. Then in 1994, Al-Alaoui [14] proposed an IIR differentiator obtained from the Simpson Integrator using the same approach. Adding a delay of half sample to the differentiator proved to be significant as it shifts the discontinuity from the Nyquist frequency to zero frequency where the magnitude was zero thereby alleviating the approximation difficulties. These designs were limited in application to the narrow-band signals. But to process narrow-band signals in diverse frequency ranges, wideband differentiators have to be designed. So it may have been more illustrative to broaden the scope of the study to wideband designs of digital differentiators.

Ngo [15], in 2006, applied the z-transform technique to the closed-form Newton-Cotes integration formula to develop a new wideband 3rd-order trapezoidal digital integrator and from this, a new wideband differentiator was designed that approximated the ideal one over the whole Nyquist frequency and compared with the existing low pass differentiator favorably. In 2007, Al-Alaoui [16] addressed a new design technique using fractional delay that controlled the magnitude and phase of integrators and differentiators to be used in applications like wave-shaping, oscillators, controls and communications that require a constant phase of 90° or -90° for differentiators and integrators respectively to overcome the phase discontinuity at Nyquist frequencies. Based on the conclusions taken from [17], a fractional delay was added to the integrator and the sampling rate was doubled; which resulted in improved magnitude and phase responses of the proposed designs. The same approach was then applied to design differentiators in which instead of adding delay, a fractional advance was realized as the ratio of two delays. But adding a fractional advance was somehow tedious as compared to adding a delay.

Gupta *et al.* [18], in 2010, addressed a class of stable wideband IIR digital integrators and differentiators of third-order by using the approach given in [12]. These designs were significant differentiators over the whole Nyquist band as compared to the existing ones. Interpolation of Schneider-Kaneshige-Groutage (SKG) integration and trapezoidal integration was employed for deriving the transfer function of the integrator which was then modified approximately to obtain the differentiator. Gupta *et al.* [19], in 2011, presented another design of integrator by performing linear interpolation between SKG, Al-Alaoui optimized 4-segment integrator rule and rectangular rule. Then after modifying the transfer function of this integrator, a suitable wideband differentiator was obtained that outperforms the previous designs in both range and accuracy. It was observed that the proposed designs perform favorably better when compared with the component differentiators used for interpolating.

Jain *et al.* [20], in 2013, introduced a family of stable IIR digital integrators and differentiators by using minimax and pole, zero and constant (PZC) optimization methods. The authors designed a class of second, third and fourth-order digital integrators by using the minimax optimization method and subsequently improved the magnitude response and group delay response using the PZC optimization method. Then differentiator was designed from the obtained integrator by modifying the transfer function. Proposed designs were tested with test signals such as triangular or square wave signals which when integrated or differentiated gave proper results compared to the previous designs that gave distorted waves. Table1 shows the comparison of various conventional techniques used to design IIR-based digital differentiators in terms of magnitude response, total absolute magnitude error, phase response, and maximum phase error metrics.

Table 1. Comparison of Conventional Techniques to design IIR-based Digital Differentiators

Reference	Technique	Order	Magnitude Response	Absolute Magnitude Response Error (%)	Phase Response	Phase Response Error(°)
Al-Alaoui, [12]	Inversion of Transfer Function of Tick Integrator	2	0.5	1	-	-
Al-Alaoui, [13]	Interpolation of Rectangular and Trapezoidal Rule	1	0.78	2	-	8.25
Al-Alaoui, [14]	Simpson Integration Rule	2	0.4	1.7	-	-
Ngo, [15]	z-transform technique to Newton-Cotes Integration Formula	3	-	5	-	12.2
Al-Alaoui, [16]	Using Fractional Delay to control the magnitudes and phases of Integrators and Differentiators	1	-	-	90	-
Gupta, [18]	Interpolation of Schneider-Kaneshige-Groutage (SKG) integration and trapezoidal integration techniques	3	-	0.14	-	-
Gupta, [19]	Linear interpolation between SKG, Al-Alaoui optimised 4-segment integrator rule and rectangular rule	3	-	2.8	-	11
Jain, [20]	Using minimax and pole, zero and constant (PZC) optimization	2 3 4	-	0.31 0.22 0.12	-	-
Cheng, [21]	Bilinear transformation	1	0.8	1	-	9.5

2.2 Design of IIR Digital Differentiator using Evolutionary Optimization Algorithms

All the above mentioned conventional methods were efficient only in case of the unimodal optimization problem. So to solve multi-modal problems, researchers moved towards nature-inspired evolutionary techniques. For this, various nature-inspired heuristic and meta-heuristic evolutionary optimization algorithms were applied to optimize the filter coefficients.

Upadhyay, [22], in 2010, presented a class of wideband recursive digital differentiators with less than 2% relative error in magnitude response using pole-zero (PZ) plot analysis. Also, the phase response of the proposed designs had comparable results with that of the existing designs. Then in 2011, Al-Alaoui [23], developed a class of IIR digital integrators, from which, a class of IIR digital differentiators was derived by applying a class of numerical integration rules. Newton-Cotes-based digital differentiators were obtained by inverting the integrator transfer functions derived by the same techniques after magnitude compensation. Also, by linear interpolation of trapezoidal and rectangular rules yielded a class of non-minimum phase designs of integrators, and from them, a stable and minimum phase Al-Alaoui first-order differentiator was derived. Then Ngo's third-order integrator and differentiator were derived along with the Pei-Hsu 2nd-order differentiator by employing fractional delays to filter. Also, 2-segment, 3-segment, and 4-segment rules were applied to design a new class of differentiators and integrators which were known to exhibit many characteristics such as low-pass design at high frequency and having lower errors. But they had a problem of pole lying on unit circle due to which stabilization was difficult. So to get over this problem, the SA optimization algorithm was employed that proved to be effective in removing all the limitations of the existing class of differentiators and integrators given in this paper.

In 2013, Al-Alaoui *et al.* [24], addressed wideband designs of digital integrators and differentiators using different optimization techniques such as Simulated Annealing (SA), Modified Fletcher and Powell optimization and Genetic Algorithm (GA) without modifying the phase response which aimed to minimize the mean absolute magnitude error over the entire frequency range as well as to keep the zeros and poles inside the unit circle. When compared to the analog counterparts, the differentiators proved to be the best first-order digital differentiators with high accuracy and efficient for multimodal optimization problems. The worth noting point was that the obtained differentiators did not rely on inverting the integrators of similar order rather they were developed on their own. Gupta *et al.* [25], in 2014, presented the 2nd, 3rd, and 4th-order designs of IIR digital differentiators after modifying the PSO algorithm. Mean squared error was optimized to a minimum by using Modified Particle Swarm Optimization (MPSO). Obtained results were having low relative magnitude errors for full-band of Nyquist frequency.

Kumar *et al.* [5], in 2015, applied a metaheuristic algorithm known as Interior Search Algorithm (ISA) to design IIR-based 2nd, 3^{rd,} and 4th-order digital differentiators; the magnitude response of which was found to be accurately approximate the ideal differentiator over the entire Nyquist frequency range. Importantly, this algorithm had to control only a single parameter and thus had a fast convergence rate, resulting in mitigation of the premature convergence and stagnation problems. Results showed that this algorithm was crucial as it surpassed the previously used algorithms for this purpose. Al-Alaoui *et al.* [26], in the same year, presented a modified Particle Swarm Optimization (PSO) technique to design a second and third-order IIR-based digital differentiator. Using the coefficients of existing filter designs, a new starting point was chosen to get converged to an optimal solution. A set of digital integrators were obtained by modifying the fitness function of PSO for a specified range of frequency. The transfer function of the resulting integrator design was inverted to get a digital differentiator which was further modified using PSO to surpass the performance.

Kumar [4], in the same year, adopted a new heuristic optimization algorithm known as the Gravitational Search Algorithm (GSA) for designing IIR differentiators of 2nd, 3rd, and 4th-order. The algorithm was based on Newton's Gravitational law of attraction, that has the advantage of its suitability for higher-order problems, to find the optimal solutions to the differentiator problem in which the agents tend to act like masses and force of attraction between these agents were evaluated from gravitational law. In analogy to that law, the agents with larger mass were served as the near optimal solutions to find the filter coefficients. It was worth-noting that PSO, GA, and SA lead to a sub-optimal solution as they suffer from premature convergence and stagnation drawbacks. On contrary, the solutions from GSA were found to be exceptional with lower magnitude and phase errors when compared to SA, PSO, GA, PZ, and segment rule discussed above.

Mahata *et al.* [27], in 2016, proposed an evolutionary algorithm called Harmony Search Algorithm (HSA) to design 1st, 2nd, 3rd, and 4th-order wideband and stable recursive digital differentiators and integrators with least absolute magnitude response error. Results of HSA were compared with that of Real-coded Genetic Algorithm (RGA), Differential Evolution (DE), and PSO, which were found to be superior to these algorithms; for they were not effective in fixing the premature convergence and stagnation problems. Aggarwal *et al.* [28], in 2017, presented 2nd, 3rd, and 4th-order IIR based digital differentiators by optimizing the L_1-error fitness function using Bat Algorithm (BA). L_1-norm of the error fitness function was disparaged to reckon the coefficients of numerator and denominator of the differentiator to corroborate the poles and zeros to be within the unit circle. The results of the proposed designs were compared with those proposed by using PSO and Real-coded Genetic Algorithm (RGA). These designs anticipated high accuracy and the relative magnitude error found to be much lower with the flat response in the wideband frequency range.

Mahata *et al.* [29], in 2018, proposed a stable and accurate design of wideband IIR digital differentiators and integrators by applying Hybrid Flower Pollination Algorithm (HFPA) optimization technique. These designs were then compared with RGA, PSO, Differential Evolution (DE), success-history-based adaptive differential evolution with linear population size reduction (L-SHADE), self-adaptive Differential Evolution (jDE) and Flower Pollination Algorithm (FPA) based designs in terms of robustness, optimization time, rate of convergence and solution quality. Lower orders of proposed designs render them to exhibit lower computational complexity and smaller memory requirements. The designs were aimed to minimize the root mean square error (RMSE) and maximum absolute magnitude error (MAME). HPFA designs were proved to perform better than all other optimization techniques based designs taken for comparison.

Goswami *et al.* [30], in 2019, presented an approach to design an Nth-order IIR digital differentiator that involved the optimization of two parameters; fractional delay and weighting variable that was optimized to approximate the frequency response of the differentiator for the frequency range of interest. Bilinear transformation and rectangular transform were fractionally interpolated and the unknown variables were optimized using GA. Proposed designs were observed to perform superior to all the existing designs in terms of magnitude response that attained mean relative magnitude error to be much lower in the complete Nyquist range. Ahmed *et al.* [31], in the same year, proposed a novel approach based on integrating a weighted L_1-norm optimization criterion with Salp Swarm Algorithm (SSA) to design minimum-phase, stable and wideband 2nd- to 4th-order IIR digital differentiators. This design was then compared, in terms of consistency, accuracy, and efficiency, with two widely used benchmark algorithms, real-coded genetic algorithm, and particle swarm optimization taking 500 maximum iterations; the results of which revealed the superiority of the proposed design in achieving the better magnitude response with a lower absolute relative error over the other approaches. To confirm the effectiveness of designs in practical applications, triangular and square waves were applied to the inputs of them.

3. Results and Discussions

In this section, the results and discussions of the magnitude response of 2nd, 3rd, and 4th-order digital differentiators based on the various optimization techniques studied in the literature have been depicted.

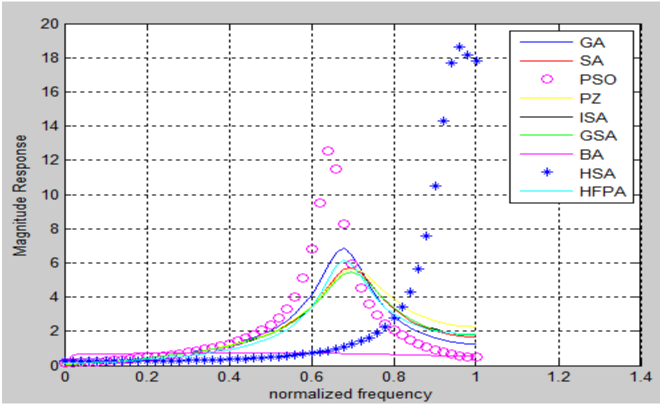

Figure 2. Comparison of Magnitude Response of 2rd-order IIR-DDs using different optimization algorithms

Figure 2 represents the comparison of the magnitude responses derived from various optimization techniques for 2nd-order digital differentiator.

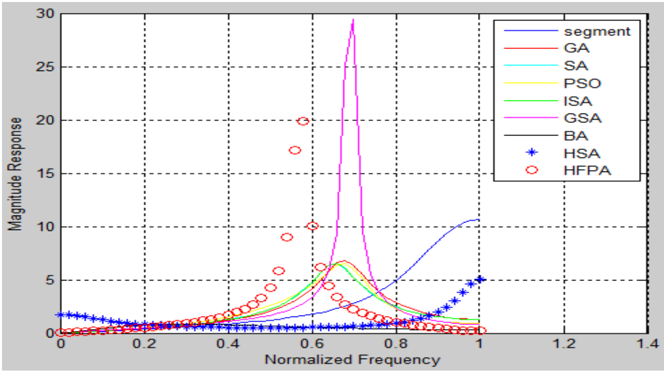

Figure 3. Comparison of Magnitude Response of 3rd-order IIR-DDs using different optimization algorithms

Figure 3 represents the comparison of the magnitude responses derived from various optimization techniques for 3rd-order digital differentiator.

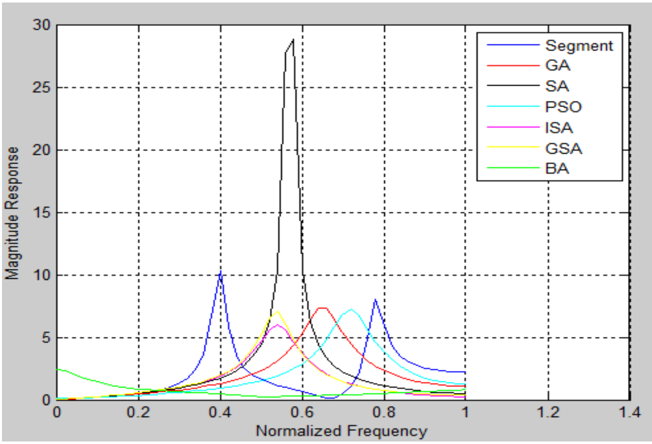

Figure 4. Comparison of Magnitude Response of 4th-order IIR-DDs using different optimization algorithms

Figure 4 represents the comparison of the magnitude responses derived from various optimization techniques for 4th-order digital differentiator.

Table2. Comparison of total absolute magnitude error and maximum phase error for different algorithms

Reference	Algorithm	Order	Total Absolute Magnitude Error	Maximum Phase Error
Upadhyay, [22]	PZ	2	4.7932	89.9102
Al-Alaoui, [23]	Segment Rule	2 3 4	154.5997 5.5590 13.0305	89.5617 269.7400 89.8869
Alaoui, Baydoun, [24]	GA	2 3 4	2.0849 1.8570 2.4899	89.9005 89.9023 89.8713
Alaoui, Baydoun, [24]	SA	2 3 4	1.5966 1.5113 1.3811	89.9092 89.9123 89.9088
Alaoui, Baydoun, [24]	Fletcher-Powell	2 3 4	10.2923 6.5313 7.0794	- - -
Gupta, [25]	PSO	2 3 4	36.6872 91.4645 3.3717	89.9983 89.7158 179.7707
Kumar, [5]	ISA	2 3 4	1.6091 1.3232 1.2287	89.9095 89.9111 99.6842
Al-Alaoui, [26]	MPSO	2 3	1.4 0.975	- -
Mahata, [4]	GSA	2 3 4	1.6021 1.4223 1.3163	89.9101 89.9037 89.9004
Mahata, [27]	HSA	2 3 4	3.6728 4.8417 4.6238	- - -
Aggarwal, [28]	BA	2 3 4	1.6054 0.3279 0.1301	- - -
Mahata, [29]	HFPA	2 3	3.16 2.46	- -
Goswami, [30]	GA	1 2	12.9665 5.5647	- -
Ahmed, [31]	SSA	2 3 4	0.1412 0.0316 0.0158	- - -

Table 2 shows the comparison of total absolute magnitude error and maximum phase error for different algorithms. In spite of the fact that nature-inspired evolutionary techniques solve multi-dimensional problems of optimization, they have a number of weak points. They show limitation in compensating control parameters. Sometimes the algorithm re-evaluates local solutions of filter coefficients and cannot reach a global minimum solution.

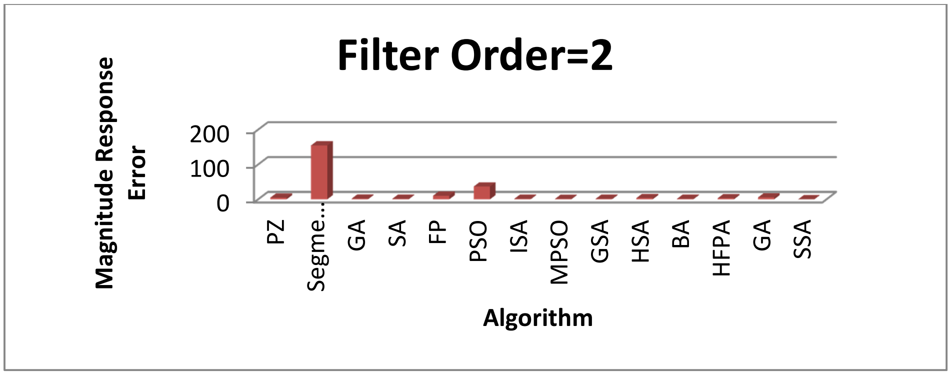

Figure 5. Comparison of magnitude response error of 2nd-order digital differentiators using different algorithms

Figure 5 shows the comparison of magnitude response error of 2nd-order digital differentiators using various algorithms proposed in the review. It is observed that segment rule presents much error while SSA has been found to present much less error.

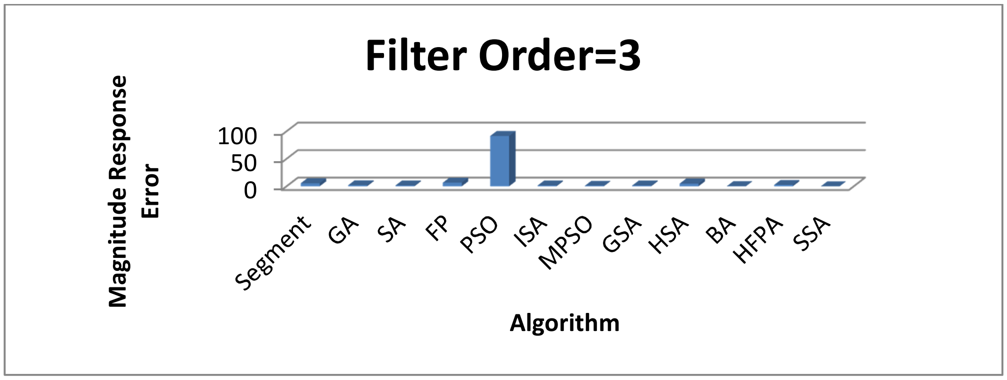

Figure 6. Comparison of magnitude response error of 3rd-order digital differentiators using different algorithms

Figure 6 represents the comparison of magnitude response error of 3rd-order digital differentiators using different algorithms. Among all PSO has more error while SSA presents less error.

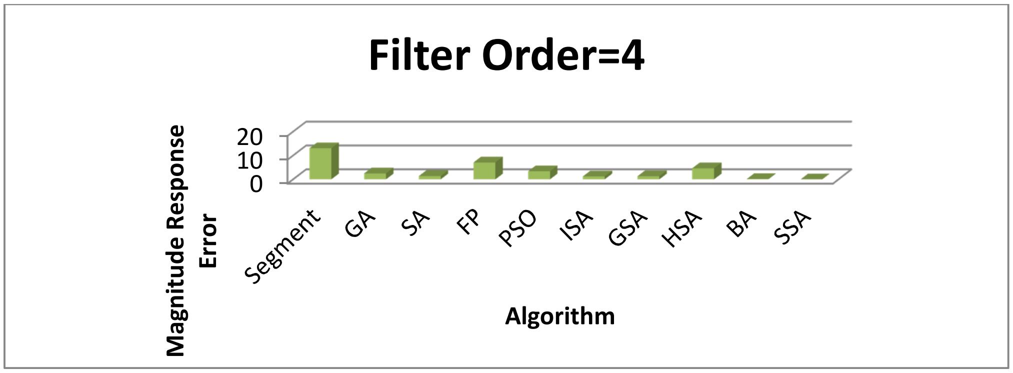

Figure 7. Comparison of magnitude response error of 4th-order digital differentiators using different algorithms

Figure 7 shows comparison of magnitude response error of 4th-order digital differentiators using different algorithms. Segment rule, Fletcher-Powell optimization algorithm, PSO, and HSA show much error but

SSA displays much less error. Fig8 shows the whole comparison of magnitude response error shown by 2nd, 3rd, ands 4th-order digital differentiator designs proposed by using various optimization algorithms applied in the survey.

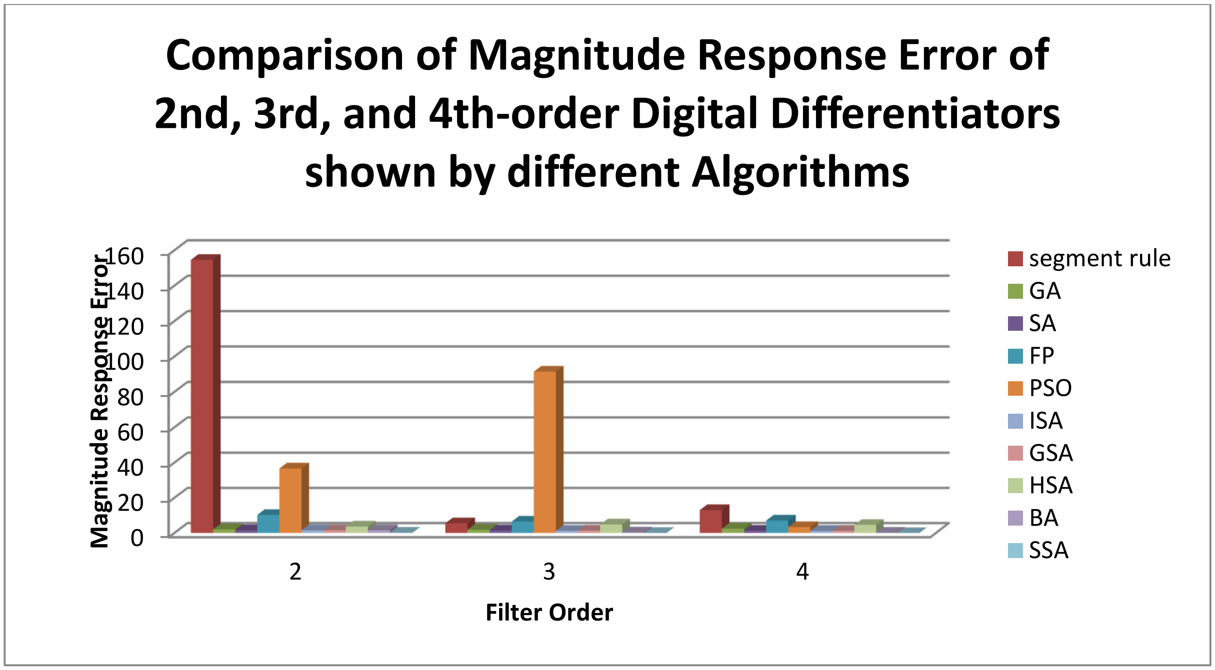

Figure 8. Comparison of magnitude response error of 2nd, 3rd, and 4th-order digital differentiators using different algorithms

From the survey, it has been observed that GA is inexpedient to find global minimum. Also, it exhibit slow convergence rate and takes undesirable time in finding optimal solution involving large search space. Problem of early convergence and stagnation has been seen in PSO due to which algorithm converges to unwanted sub-optimal solutions. SA shows problem in telling whether the optimal solution has arrived or not. There is diversity loss problem in getting hold of the masses of the objects in GSA. But it has been observed from the survey that SSA performs superior to all other algorithms in terms of magnitude response when magnitude response error was measured. It has been found to present almost linear response over wideband frequency region.

4. Conclusion

The aspiration of this survey has been accomplished by giving a satisfactory overview of the research methodologies adopted to design IIR-based digital differentiators by various researchers. Various conventional as well as evolutionary optimization techniques that are applied by them in order to meet the filter requirements have been analysed satisfactorily. Among various natural selection and evolution-based techniques, Salp Swarm Algorithm has been found to be the best in terms of reducing the magnitude response error to least. However there is always a room for further improvement in the optimization of parameters. So these can be further optimized using other hybrid optimization techniques in future. Also it has been analysed that the lower orders of the proposed designs are suitable for real-time signal processing applications due to their lesser computational complexity and reduced memory requirements. Higher-order designs have a problem of multivariable optimization; so defining the distribution of minimum solutions is quite challenging. In future, this problem can be taken into consideration by using evolutionary optimization techniques as well.

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