ON CHROMATIC CORE SUBGRAPH OF CORONA GRAPH AND JOIN GRAPHS WHICH ADMITS JOHAN COLOURING

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Abstract

If distinct colors represent distinct technology types at the vertices of a simple graph in accordance to minimum proper coloring , then a disaster recovery strategy could rely on an answer to the question: " What is the maximum destruction, if any, the graph can undergo whilst ensuring that atleast one of each technology type remain, in accordance to minimum proper coloring of the remaining induced subgraph". The notion of Chromatic core subgraph H of given graph G was introduced in our paper "On Chromatic Core Subgraph of Simple Graph" which answer to the stated problem. In this paper we have found Chromatic core subgraph of graph operations such as Join of graphs and Corona product of graphs of path, cycles and complete graphs which admit Johan colouring. Ideas for further research are given.

Keywords: Chromatic Core Subgraph, Corona Product of Graphs, Johan Coloring, Johan Number, Join of graphs, Rainbow neighbourhood and Rainbow neighbourhood number.

1 Introduction

For general notation and concepts in graphs and digraphs see [1, 3, 7]. We will write that a graph G has order v (G) = $n \geq 1$ and size ε(G) = $p \geq 0$ with minimum and maximum degree δ(G) and Δ(G), respectively. Unless mentioned otherwise, all graphs Gare finite, undirected simple graphs. We recall that if C = $\{c_1, c_2 . . . c_l\}$ and l sufficiently large, is a set of distinct colours, a proper vertex colouring of a graph G denoted $\varphi : V(G) \mapsto C$ is a vertex colouring such that no two distinct adjacent vertices have the same colour. The cardinality of a minimum set of colours which allows a proper vertex colouring of G is called the chromatic number of G and is denoted $\chi(G)$.

2 Chromatic Core Subgraph and Johan Colouring

For a graph G its structural size is measured by its structor index denoted and defined as, si (G) = v (G) + ε(G). We say that the smaller of graphs G and H is the graph satisfying the condition, min {si (G), si (H)}. If si (G) = si (H) the graphs are of equal structural size but not necessarily isomorphic. A straight forward example is the path, P_4 and the star graph, S_3.
The notion of a rainbow neighbourhood of a graph G with a chromatic colouring C has been defined in [6] as the closed neighbourhood N[v] of a vertex $v \in$ V (G) which contains at least one colored vertex of each colour in the chromatic colouring C of G. Motivated by this study, a new graph colouring, namely Johan colouring is admitted to graphs as follows.

1

For a finite, undirected simple graph G of order v (G) = n ≥ 1 a **Chromatic core subgraph** H is a smallest induced subgraph H (smallest in respect of si (H)) such that, $\chi(H) = \chi(G)$.

A proper k-colouring C of a graph G is called the **Johan colouring** or **the J-colouring** of G if C is the maximal colouring such that every vertex of G belongs to a rainbow neighbourhood of G. A graph G is J-colorable if it admits a J-colouring. The **J-colouring number** of a graph G, denoted by J (G), is the maximum number of colours in a J-colouring of G.

3 Chromatic Core Subgraph and Johan Parameters of certain classical graphs

This section begins with the chromatic core subgraph for certain classical graphs and trivial observations related to it.

The following proposition and the theorem are the proofs for the conjectures 2.1.1 and 2.1.2 in [5]

Proposition 3.1 An acyclic graph G has J (G) =2 and it has P_2 as a chromatic core subgraph.

(i) An even cycle C_n, $n \geq 4$ has J (G) =2 and it has P_2 as a chromatic core subgraph.
(ii) The complete graph K_n, $n \geq 1$ has J (G) =n and it has K_n as its unique chromatic core subgraph.
(iii) For C_3 and Odd wheel W_n, $n \geq 5$ has J (G) =3 and it has C_3 as a chromatic core subgraph.

Proof:

Part (i) Since, any acycle graph can be coloured using two colours the chromatic core subgraph is P_2 and J(G) is two, its obivious by the definition of Johan colouring. Part (ii) In the case of even cycles the order is even say, n and the proof is similar as in (i). In the case of odd cycles, the proof is similar as in the above cases. (i.e)., As the order of odd cycles is n+1 the colours used to colour the vertices increases by 1 to part (ii). In Part (iii) proof is obvious by the definition of Complete graph and Johan colouring.

Corollary 3.2 For odd cycle C_n, $n \geq 3$ and Even wheel W_n, $n \geq 4$ the following are equivalent:

(i) The Johan colouring does not exist.
(ii) Johan number does not exist.

Proof:

In the case of Odd Cycle graph G with odd order and if the vertices are properly coloured then all the vertices of the odd cycle does not belong to rainbow neighbourhood of G. Hence, it is obvious that Johan colouring does not exist implies Johan number does not exist.

Remark[5] The above corollary holds for graphs with pendant vertices.

2

4. Chromatic Core Subgraph admits Johan colouring of Join of Graphs

This subsection begins with results on Join of some graphs. The **Join** of two graphs G and H denoted as G + H is obtained by taking a copy of graphs G, H and adding the edges of the complete bipartite graph between the vertices V (G) and V (H).Note that unless mentiosned otherwise, only finite, undirected connected simple graphs will be considered.

Theorem 4.1 For graphs G and H with Chromatic core subgraphs G' and H' respectively, the Chromatic core subgraph of G+H is G'+H'. For G and H can be C_3, Even cycle, path, complete graphs.

Proof:

Let $V(G)=(v_1,v_2,....v_n)$ and $V(H)=(u_1,u_2,....u_m)$ then G +H is a complete bipartite graph. There is an edge between every vertex of G to every vertex of H and vice versa also occurs. Proposition 2.1.1 gives the Chromatic core subgraph G' and H'. On observing the graphs G+H and G'+H' we conclude the result that Chromatic core subgraph of G+H is G'+H'.

The following Corollary behaves different to the theorem 4.1.

Corollary 4.2 The Chromatic core subgraph of $P_n + C_3$ is $P_n + C_3$.

5. Chromatic core Subgraph admits Johan Colouring of Corona Product of Graphs

In respect of the corona G⨀H the following results holds. Recall that G⨀H is obtained by taking one copy of graph G and order G say, n copies of graph H and for each vertex *v* in G then construct (add), v + H.

Theorem 5.1 Let *G* and *H* be two graphs the chromatic core sub graphs of G⨀H is C_3 whenever *G* and *H* can be path or even cycle.

Proof:

Let *G* and *H* be any graph which can be a path or an even cycle. Let $V(G)=(v_1,v_2,....v_n)$ and $V(H)=(u_1,u_2,....u_m)$, For each vertex v_i of *G* we take a copy of *H* say H_i. The corona graph G⨀H admits Johan colouring and its chromatic number number is $\chi=3$.The Chromatic core sub graph of G⨀H is C_3, for any path or an even cycle.

Theorem 5.2 Let K_m and *G* be two graphs the chromatic core sub graphs of is

$$K_m \odot G = \begin{cases} K_3 & if \quad m \le 2 \\ K_m & if \quad m > 2 \end{cases}$$ where, G be path or even cycle.

Proof:

Let K_m be a complete graph and *G* be a path or even cycle.

Case 1: When $m \le 2$

The proof is similar to case 2

Case 2: When m > 2

Let $V(K_m) = (v_1, v_2, ...v_m)$ and $V(G) = (u_1, u_{2,} ...u_n)$.For each vertex of K_m we take a copy of G says that $G_1, G_2 ... G_m$. The vertices $v_1, v_2, ...v_m$ of G form an edge with every vertex of $G_1, G_2 ... G_m$ respectively. On Johan colouring the obtained $K_m \odot G$ we get $\chi = m$ and Chromatic core sub graph of $K_m \odot G$ is K_m, Since the chromatic number of chromatic core sub graph of $K_m \odot G$ is m.

Example 5.3

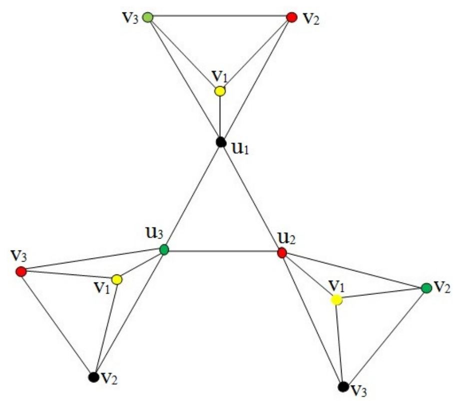

Figure 3.1: corona product of $C_3 \odot C_3$

Theorem 5.4 Let G and K_n be two graphs the chromatic core sub graphs of $G \odot K_n = K_{n+1}$ for G be an any path or any even cycle or any n-complete graph.

Proof:

Let G be a any path or any even cycle or n-complete graph and K_n be a complete graph.

Let $V(G) = (v_1, v_2, ...v_m)$ and $V(K_n) = (u_1, u_2, ...u_n)$. For each vertex of G we take a copy of K_n says that $K_1, K_2....K_n$. The vertices of G form an edge with every vertex of $K_1, K_2....K_n$ respectively. On Johan colouring the obtained $G \odot K_n$ we get $\chi = n+1$ and Chromatic core sub graph of $G \odot K_n$ is K_{n+1}, Since the chromatic number of chromatic core sub graph of $G \odot K_n$ is 3.

Theorem 5.5 Let K_m and K_n be two graphs the chromatic core sub graphs of is

$$K_m \odot K_n = \begin{cases} K_{n+1} & \text{if } m \leq 2 \\ \left.\begin{array}{ll} K_{n+1} & \text{if } n \geq m-1 \\ K_m & \text{if } n < m-1 \end{array}\right\} & m > 2 \end{cases}$$

Proof:

Let K_m and K_n be both complete graph.

4

Case 1: When m≤ 2

Let $V(K_m)=(v_1,v_2,....v_m)$ and $V(K_n)=(u_1,u_2,....u_n)$. For each vertex of K_m we take a copy of K_n says that $K_1,K_2....K_n$ respectively. The vertices $v_1,v_2,....v_m$ of K_m form an edge with every vertex of $K_1,K_2....K_n$ respectively. On Johan colouring the obtained $K_m \odot K_n$ we get $\chi = n+1$. Chromatic core sub graph of $K_m \odot K_n$ is K_{n+1}, since the chromatic number of chromatic core sub graph of $K_m \odot K_n$ is $n+1$.

Case 2 : When m > 2 and if $n \geq m-1$

Let $V(K_m)=(v_1,v_2,....v_m)$ and $V(K_n)=(u_1,u_2,....u_n)$ .For each vertex of K_m we take a copy of K_n says that $K_1,K_2....K_n$ respectively. The vertices $v_1,v_2,....v_m$ of K_m form an edge with every vertex of $K_1,K_2....K_n$ respectively. On Johan colouring the obtained $K_m \odot K_n$ we get $\chi = n+1$. Chromatic core sub graph of $K_m \odot K_n$ is K_{n+1}, since the chromatic number of chromatic core sub graph of $K_m \odot K_n$ is $n+1$.

Case 3: When $m > 2$ and if $n< m$-1

Let $V(K_m)=(v_1,v_2,....v_m)$ and $V(K_n)=(u_1,u_2,....u_n)$ .For each vertex of K_m we take a copy of K_n says that $K_1,K_2....K_n$ respectively. The vertices $v_1,v_2,....v_m$ of K_m form an edge with every vertex of $K_1,K_2....K_n$ respectively. On Johan colouring the obtained $K_m \odot K_n$ we get $\chi = m$. Chromatic core sub graph of $K_m \odot K_n$ is K_m, since the chromatic number of chromatic core sub graph of $K_m \odot K_n$ is m.

6 Conclusion:

The paper introduced the notion of a chromatic core subgraph in respect of Johan colouring. The field of research can be developed by generalizing the notion to edge colouring and other derivative colorings such as local colouring, dynamic colouring, co-colouring, Grundy colouring, harmonious colouring, complete colouring, exact colouring, star colouring and others, offers a wide scope.

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